QxQ Complex Numbers Cheat Sheet

Notation

Imaginary unit = √-1

$$z = x + iy = re^{i\varphi}$$

Phase

Real part

Imaginary part

Magnitude

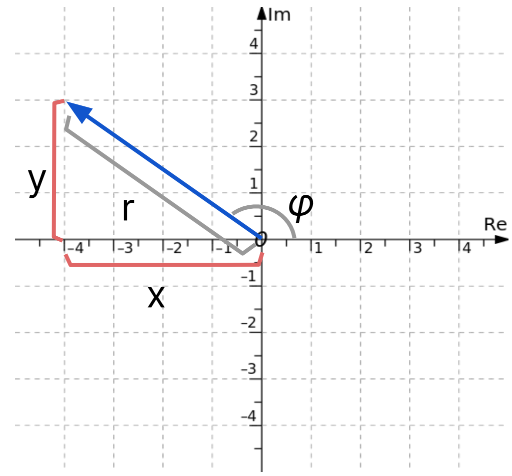
Rectangular form

Polar form

So we write that:

$x = \mathrm{Re}[z]$ and $y = \mathrm{Im}[z]$

Plotting



Convenient equation:
$zz^* = |z|^2$

Operations

Magnitude: $|z| = r = \sqrt{(x^2+y^2)}$ (remember the Pythagorean theorem!)

Phase: $\varphi = \tan^{-1}(y/x)$ (same as finding the angle in polar coordinates)

Complex conjugate: $z^* = x - iy = re^{-i\varphi}$ (flip the sign of the imaginary part)

Addition: $z_1 + z_2 = (a + bi) + (c + di) = (a + c) + (b + d)i$ (add the real and imaginary parts from each number together)

Multiplication: $z_1z_2 = (a + bi) \times (c + di) = (ac - bd) + (ad + bc)i$ (just like multiplying binomials)

$= r_1e^{i\varphi 1} \times r_2e^{i\varphi 2} = r_1r_2e^{i(\varphi 1 + \varphi 2)}$ (just like normal multiplication of exponential expressions)

Converting: from rectangular to polar

1. Find the complex magnitude $|z| = r = \sqrt{(x^2+y^2)}$
2. Find the complex phase angle $\varphi = \tan^{-1}(y/x)$
3. Put it in polar form: $z = re^{i\varphi}$

from polar to rectangular

1. Find the real part: $x = r\cos\varphi$
2. Find the imaginary part: $y = r\sin\varphi$
3. Put it together: $z = x + iy$

Euler's Equation

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

What this means:

1. $e^{i\varphi}$ (for any φ) is on the unit circle when plotted in the complex plane
2. Since we can write any set of numbers x and y as $x=r\cos\varphi$ and $y=r\sin\varphi$, every complex number can be written as $z = x+iy = r(\cos\varphi + i\sin\varphi) = re^{i\varphi}$