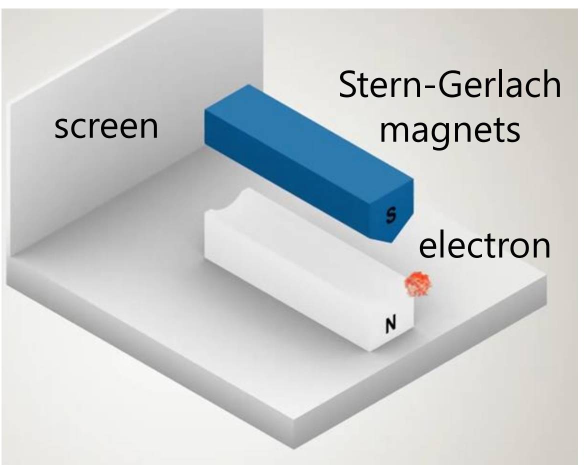
Quantum Mechanics Cheatsheet

Postulates of Quantum Mechanics



The Stern-Gerlach experiment
Measurement of an electron's spin (intrinsic magnetic moment) along a given direction

1. A quantum state is represented by its wavefunction $|\psi\rangle$

Superposition: In addition to the computational basis (or Z-basis) states $|0\rangle$ and $|1\rangle$, a (two − level) quantum state can be in any linear combination of the basis states $\alpha|0\rangle + \beta|1\rangle$ where $\alpha, \beta \in \mathbb{C}$

We will consider an equal superposition for the spin of an electron, $|\boldsymbol{\psi}\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

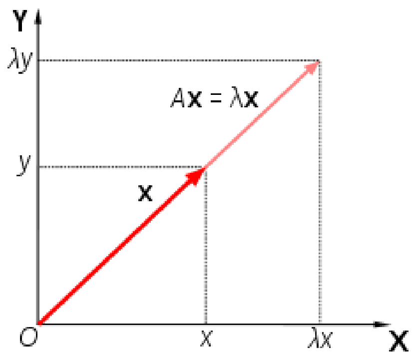
2. For any observable, there is a corresponding *operator* in quantum mechanics

In the SG experiment, the observable is **spin**. In the experiment, we align our magnets in the direction along which we wish to measure the spin

Observable	Operator	Matrix
Spin along z-axis	Z or σ_z	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Spin along x-axis	X or σ_X	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
Spin along y-axis	Y or σ_Y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

3. The only possible outcomes of measuring an observable are the eigenvalues of its operator

To obtain the possible outcomes of measurement in the SG experiment, we find the eigenvalues of the operator matrices. We will consider the measurement along the z-axis. Hence, the possible outcomes of our experiment are 1 and -1 (no matter what the initial spin of the electron is!)



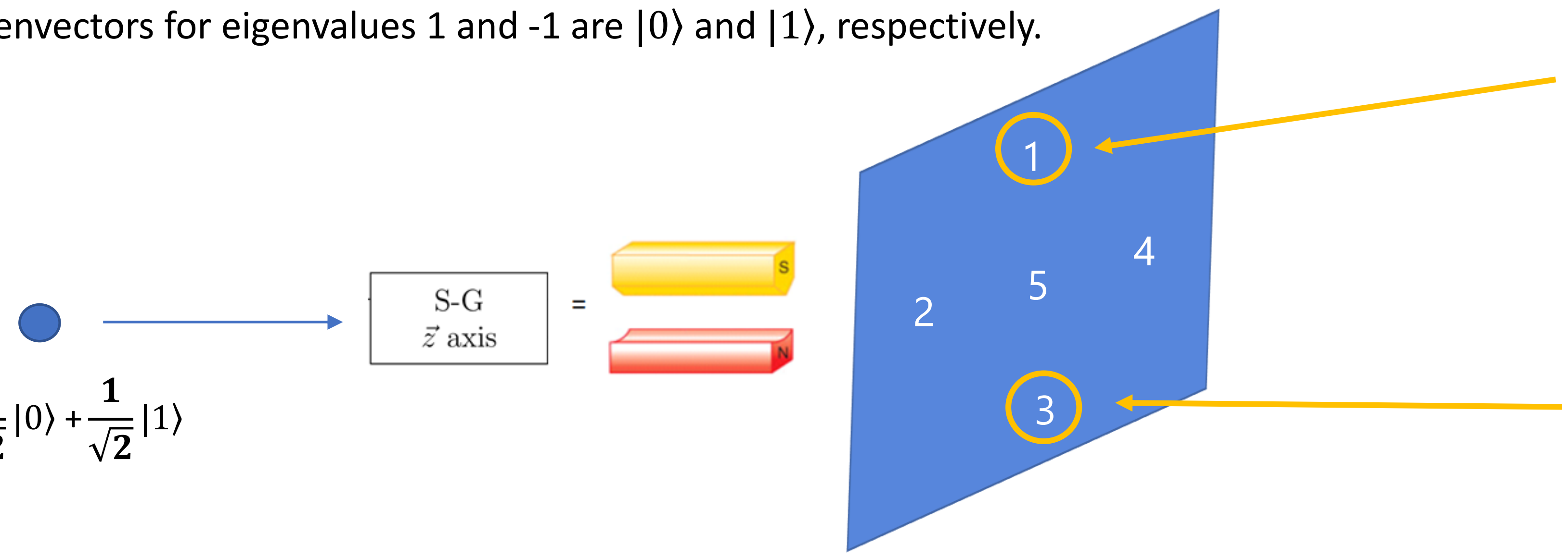
If a vector is only scaled, and not rotated, when a matrix is applied to it, it is an ***eigenvector*** *(for this matrix). The factor by which the vector is scaled is the corresponding* ***eigenvalue****.*

Here, ***x*** *is an eigenvector of A. The corresponding eigenvalue is λ.*

Operator Matrix	Eigenvalues and eigenvectors
$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$1: \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $-1: \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $-1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ and $-1: \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

4. The probability of measuring an eigenvalue is the modulus squared of the inner product of its corresponding eigenvector with the initial state.

We can calculate the probability of obtaining each of the possible outcomes using this postulate. The corresponding eigenvectors for eigenvalues 1 and -1 are $|0\rangle$ and $|1\rangle$, respectively.



$|\boldsymbol{\psi}\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

Probability of getting 1: $P(1) = |\langle \mathbf{0}|\boldsymbol{\psi}\rangle|^2 = \frac{1}{2}$

Probability of getting -1: $P(-1) = |\langle \mathbf{1}|\boldsymbol{\psi}\rangle|^2 = \frac{1}{2}$

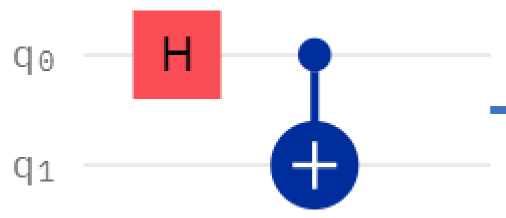
5. After measurement, the quantum state *collapses* to the eigenvector corresponding to the observed eigenvalue.

If the outcome is 1, then we know *for sure* that the electron's spin is in state $|0\rangle$ after the measurement. Similarly, if the outcome is -1, then we know that the spin is in state $|1\rangle$.

6. The time evolution of a quantum state can be modeled by a matrix. To obtain the state after the evolution, we multiply the matrix to the initial state.

Entanglement

When the state of multiple qubits cannot be described independently of each other, even when they are separated by a large distance!



$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

This state *cannot* be written as the tensor product of two single qubit states. That's why the state of these two-qubits cannot be described independently!

Bell States

$$|\boldsymbol{\phi}^+\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(|00\rangle + |11\rangle)$$

$$|\boldsymbol{\phi}^-\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(|00\rangle - |11\rangle)$$

$$|\boldsymbol{\psi}^+\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(|01\rangle + |10\rangle)$$

$$|\boldsymbol{\psi}^+\rangle = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(|01\rangle - |10\rangle)$$