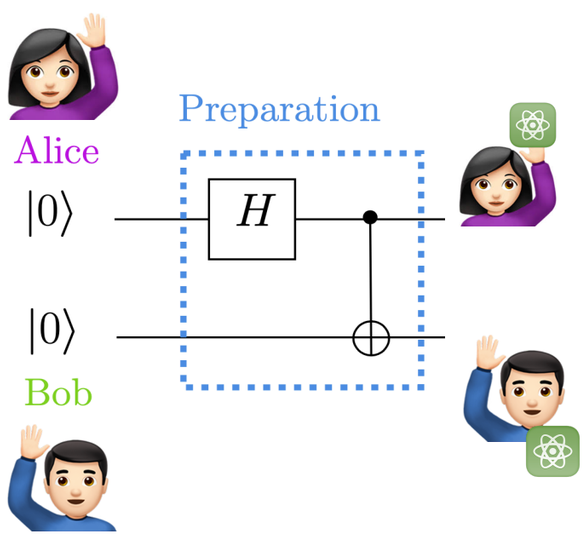
Superdense Coding Quantum Teleportation

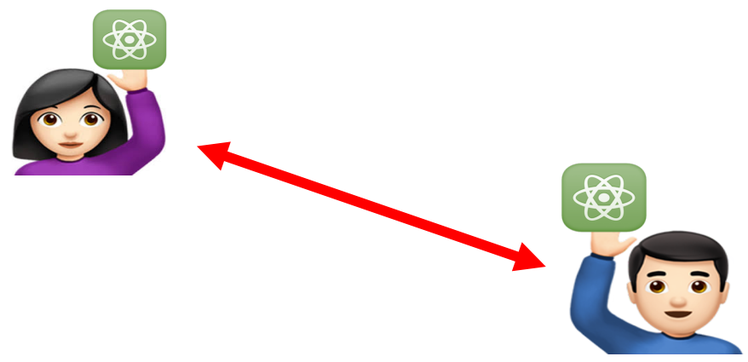
Step 1: Create and split an **entangled** bell state



$$\frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

We are using left indexing in this document. That means qubit 0 is always the leftmost qubit in any state.

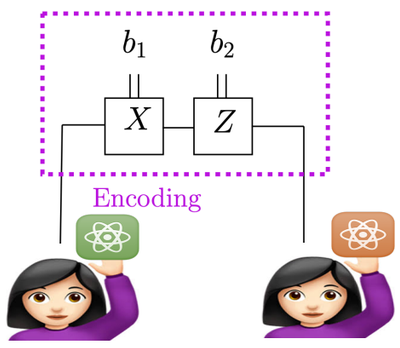
Step 2: Travel far away



Don't forget: Steps 1 and 2 are the same for both protocols!

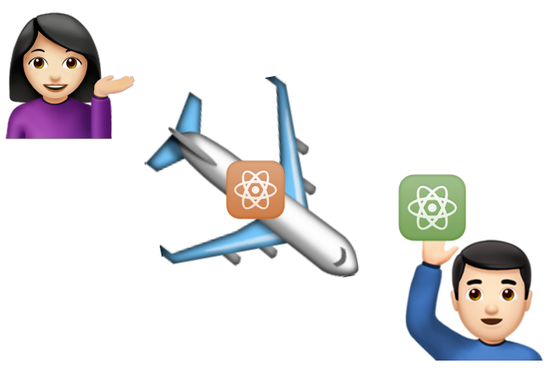
Superdense Coding

Step 3: Alice **encodes** her classical message into the qubit state

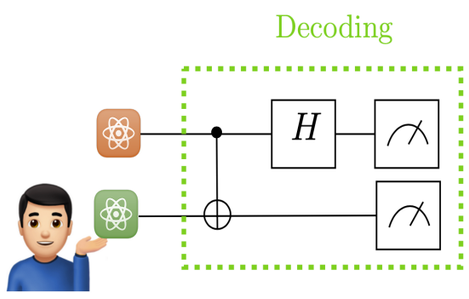


Classical message		Bell state	Alice's operations	Encoded state
0 (00)	$b_1 = 0$, $b_2 = 0$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$		$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$
1 (01)	$b_1 = 1$, $b_2 = 0$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	X	$\frac{\|10\rangle + \|01\rangle}{\sqrt{2}}$
2 (10)	$b_1 = 0$, $b_2 = 1$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	Z	$\frac{\|00\rangle - \|11\rangle}{\sqrt{2}}$
3 (11)	$b_1 = 1$, $b_2 = 1$	$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	X Z	$\frac{\|10\rangle - \|01\rangle}{\sqrt{2}}$

Step 4: Alice (physically) **sends** her qubit to Bob over a quantum channel



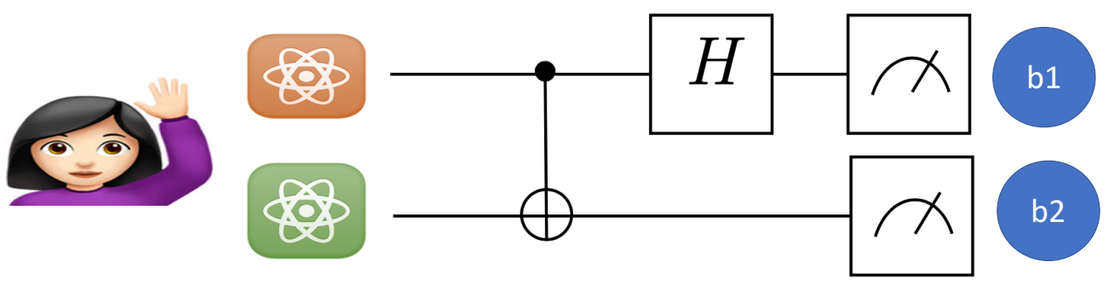
Step 5: Bob **decodes** the qubits using a bell measurement to retrieve Alice's classical message



$\frac{\|00\rangle + \|11\rangle}{\sqrt{2}}$	H	$\|00\rangle$	0 (00)
$\frac{\|10\rangle + \|01\rangle}{\sqrt{2}}$	H	$\|01\rangle$	1 (01)
$\frac{\|00\rangle - \|11\rangle}{\sqrt{2}}$	H	$\|10\rangle$	2 (10)
$\frac{\|10\rangle - \|01\rangle}{\sqrt{2}}$	H	$\|11\rangle$	3 (11)

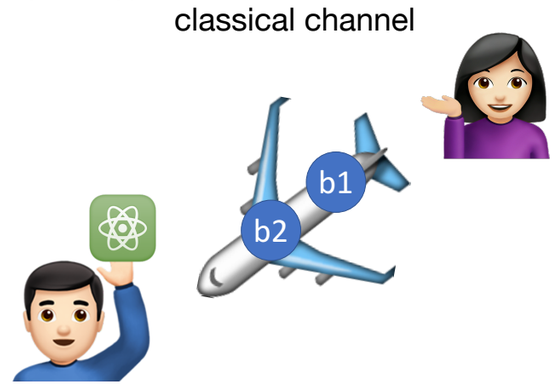
Quantum Teleportation

Step 3: Alice performs a bell **measurement** on the qubit she wants to teleport and her half of the entangled bell state.



State to be sent	Measurement result	Bob's Qubit State
$\alpha\|0\rangle + \beta\|1\rangle$	$\|00\rangle$	$\alpha\|0\rangle + \beta\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\|01\rangle$	$\alpha\|1\rangle + \beta\|0\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\|10\rangle$	$\alpha\|0\rangle - \beta\|1\rangle$
$\alpha\|0\rangle + \beta\|1\rangle$	$\|11\rangle$	$\alpha\|1\rangle - \beta\|0\rangle$

Step 4: Alice (physically) **sends** the measurement results to Bob over a classical channel



Step 5: Depending on the classical bits received, Bob corrects his qubit into the qubit that was meant to be teleported!

Classical bits	Bob's State	Bob's operations	Bob's Final State
00	$\alpha\|0\rangle + \beta\|1\rangle$		$\alpha\|0\rangle + \beta\|1\rangle$
01	$\alpha\|1\rangle + \beta\|0\rangle$	X	$\alpha\|0\rangle + \beta\|1\rangle$
10	$\alpha\|0\rangle - \beta\|1\rangle$	Z	$\alpha\|0\rangle + \beta\|1\rangle$
11	$\alpha\|1\rangle - \beta\|0\rangle$	X Z	$\alpha\|0\rangle + \beta\|1\rangle$