INTRO TO QUANTUM COMPUTING

LECTURE #R1

ALICE IN SINELAND: TRIG REVIEW

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10/17/2020

TOPICS COVERED

"Trigonometry" is the worst name in mathematics

- What are sin, cos, and tan?
- Radians and degrees
- Special Angles
- Useful trig identities
- sineland 1: coordinate-systems

  polar

  spherical
- sineland 2: waves

OBJECTIVES

- Feel comfortable with trigonometrical functions.
- Be able to use identities to simplify/covert expressions.
- Be familiar with all parts of sineland discussed here.
- Have fun!

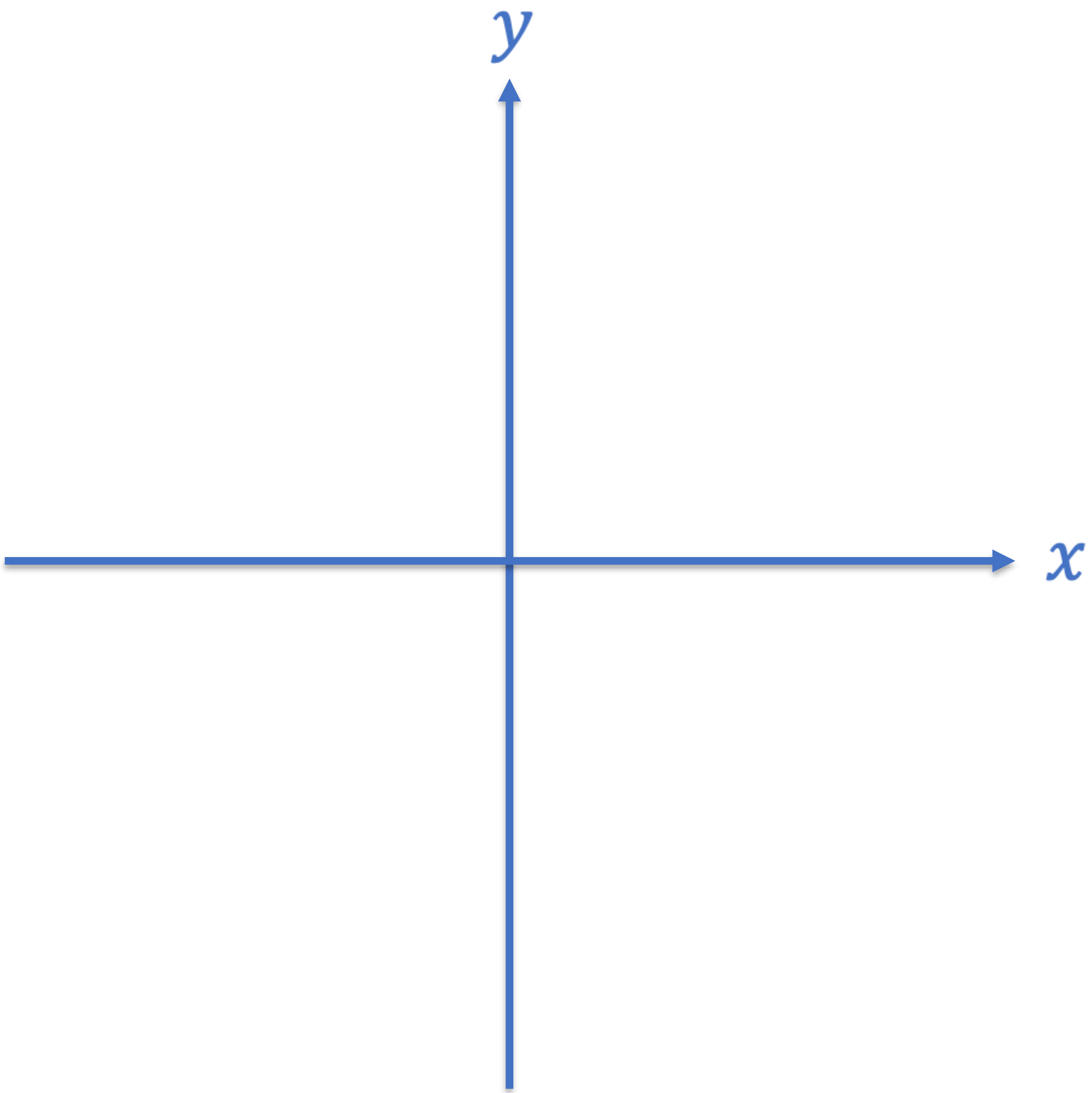
What are sin, cos, and tan?

UNIT CIRCLE

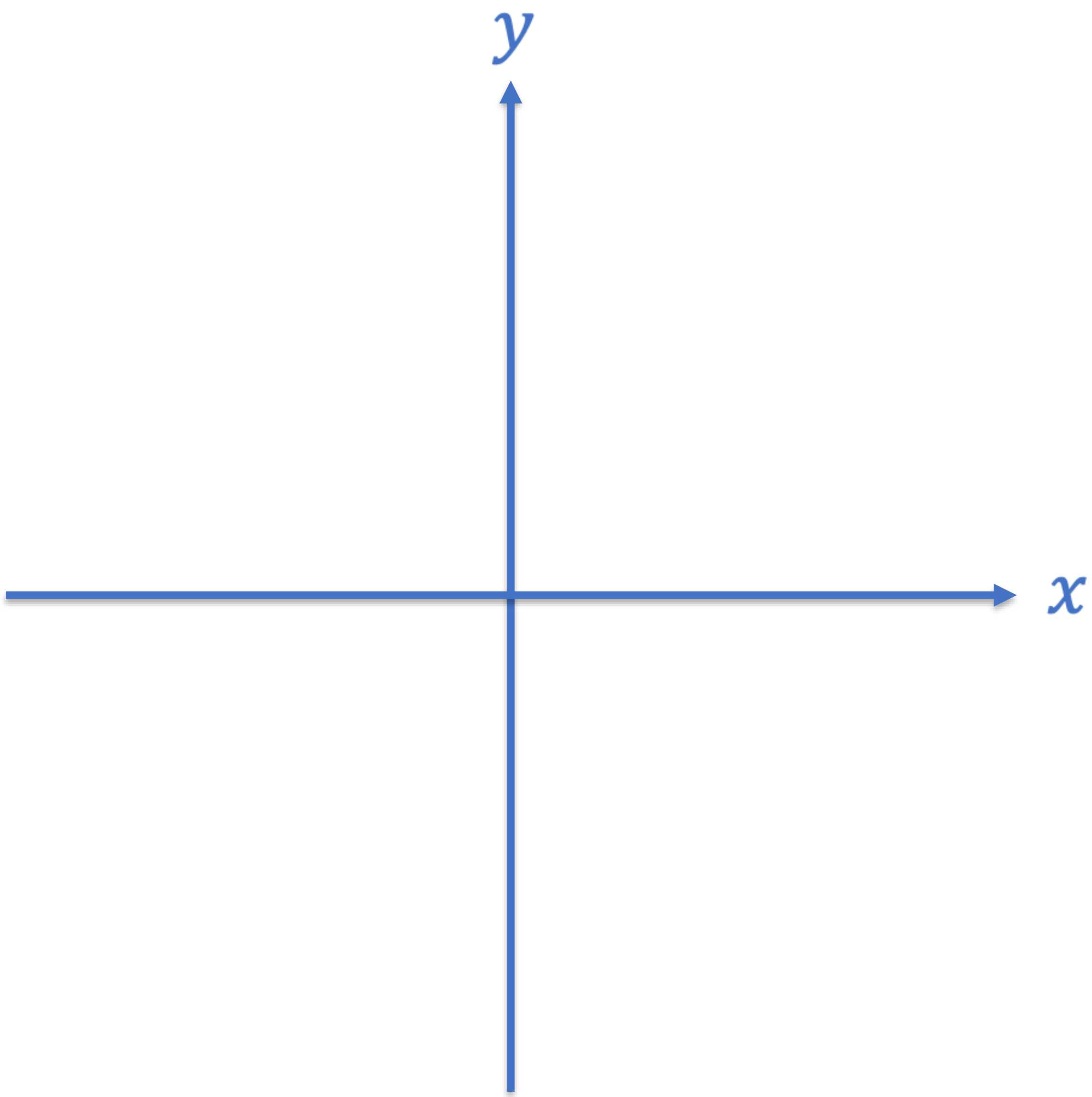
Unit Circle Definition:

1. **It is a circle…**
2. **…centered at the origin…**
3. **…with radius 1.**

Equation of a circle:

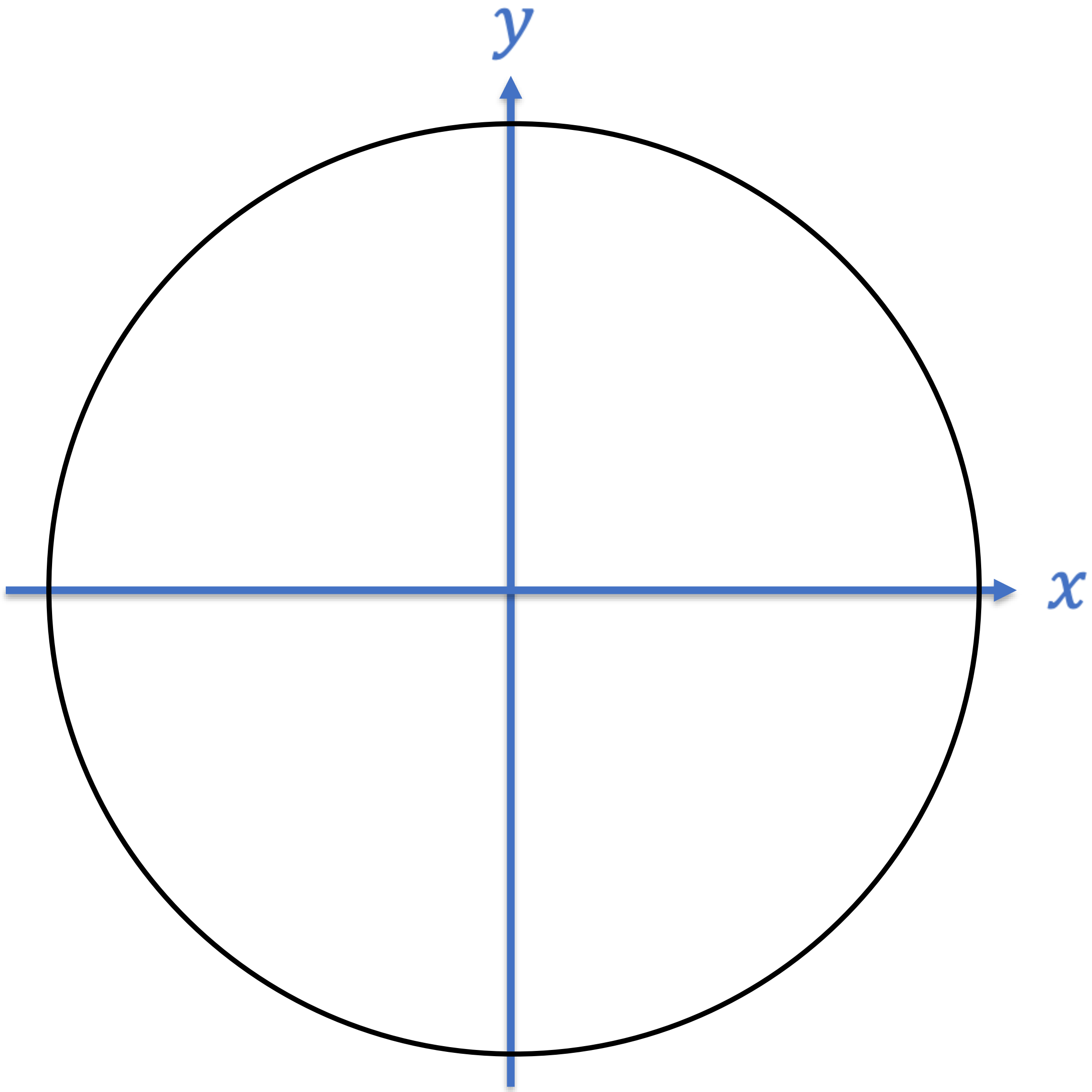


The point $P = \left(\frac{3}{5}, b\right)$ lies on the unit circle. Solve for b.

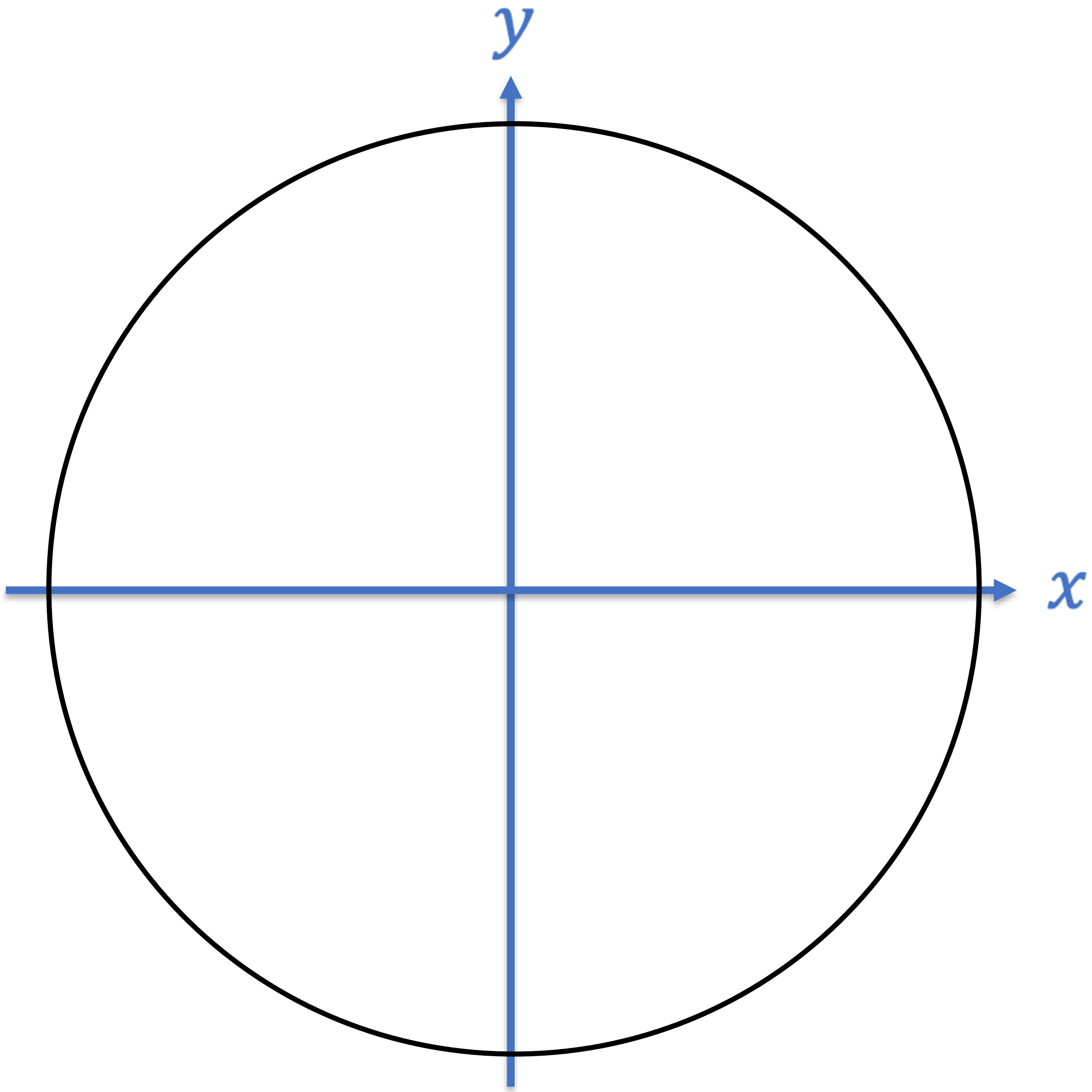


Equation of a circle:

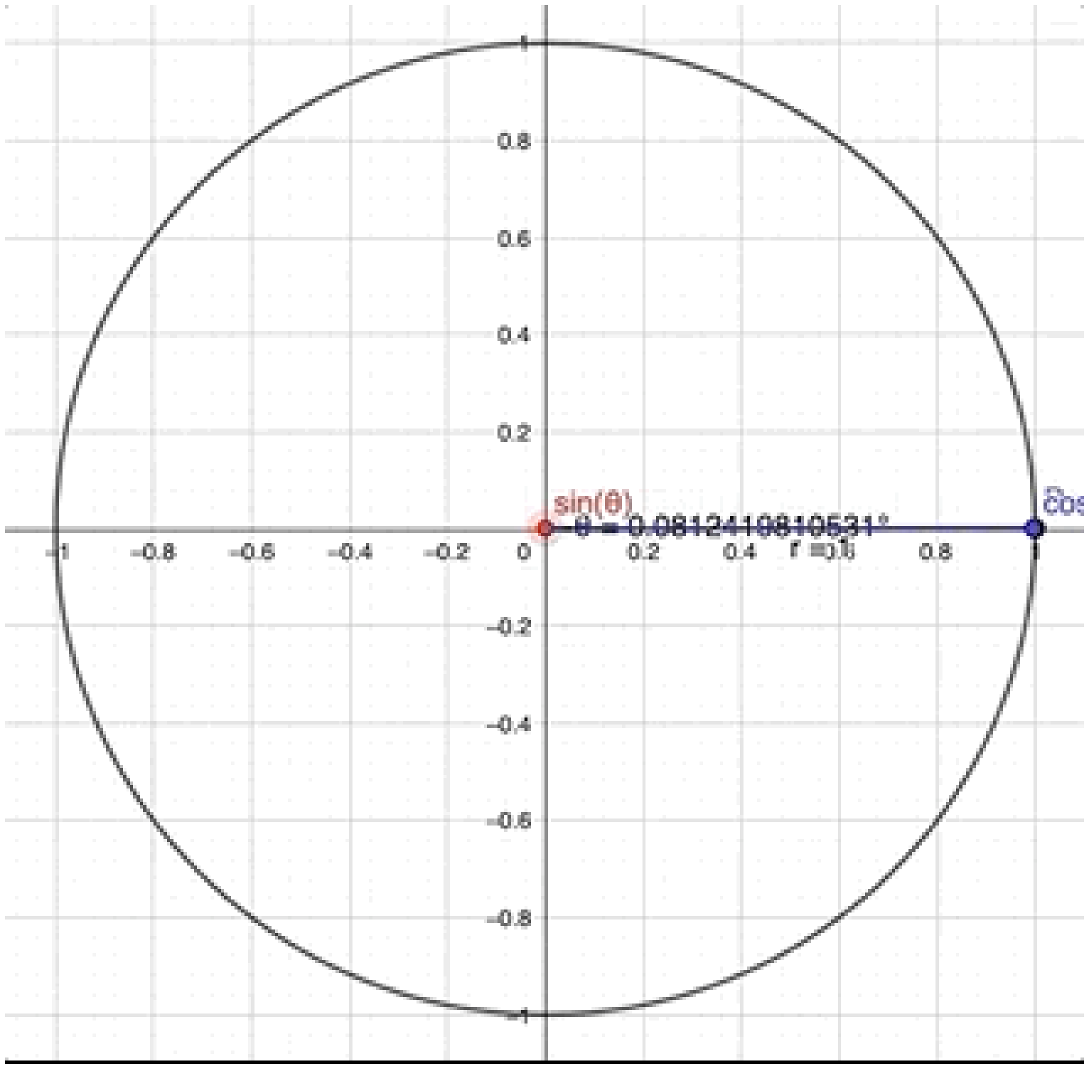
SINE AND COSINE

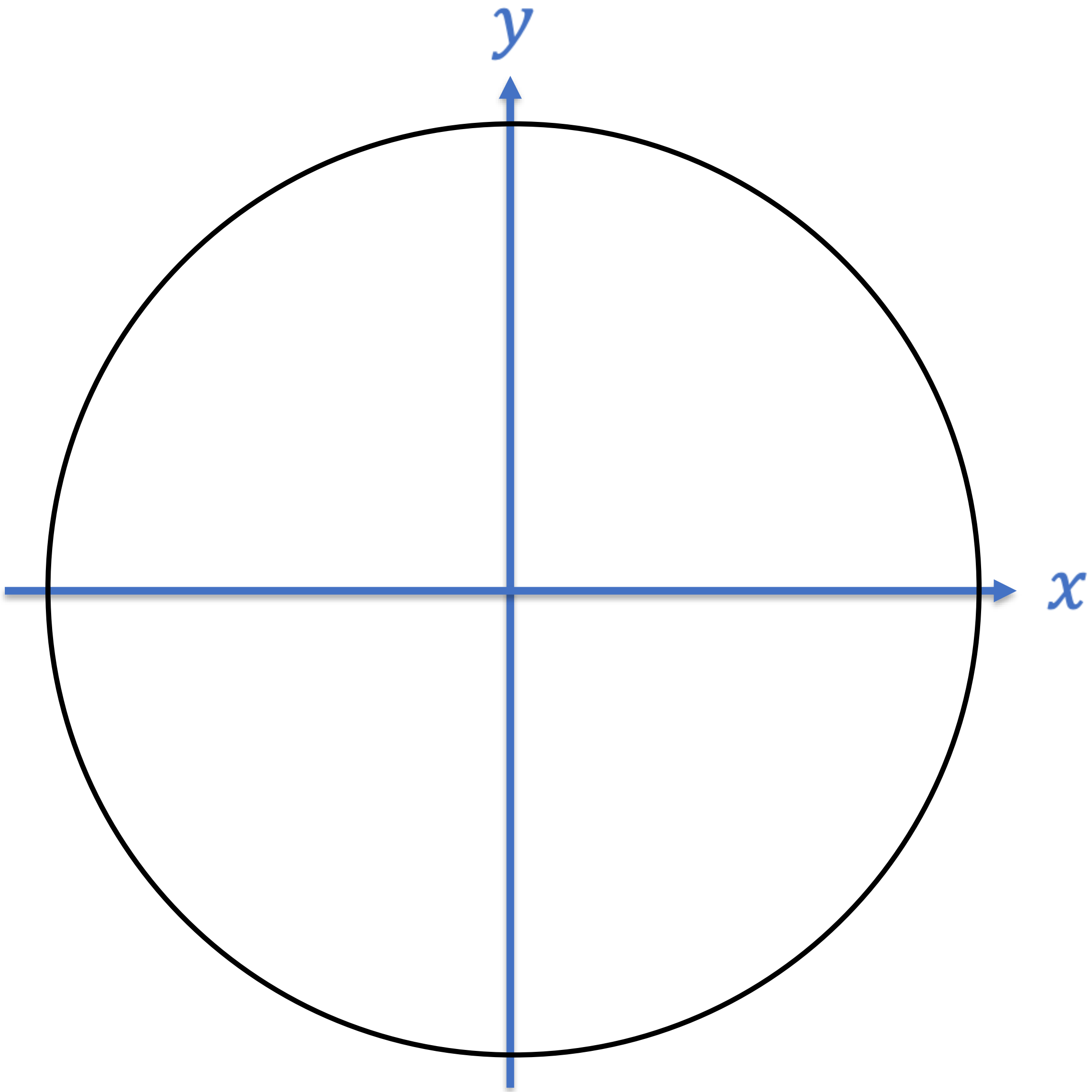


TAN AND INVERSE TAN

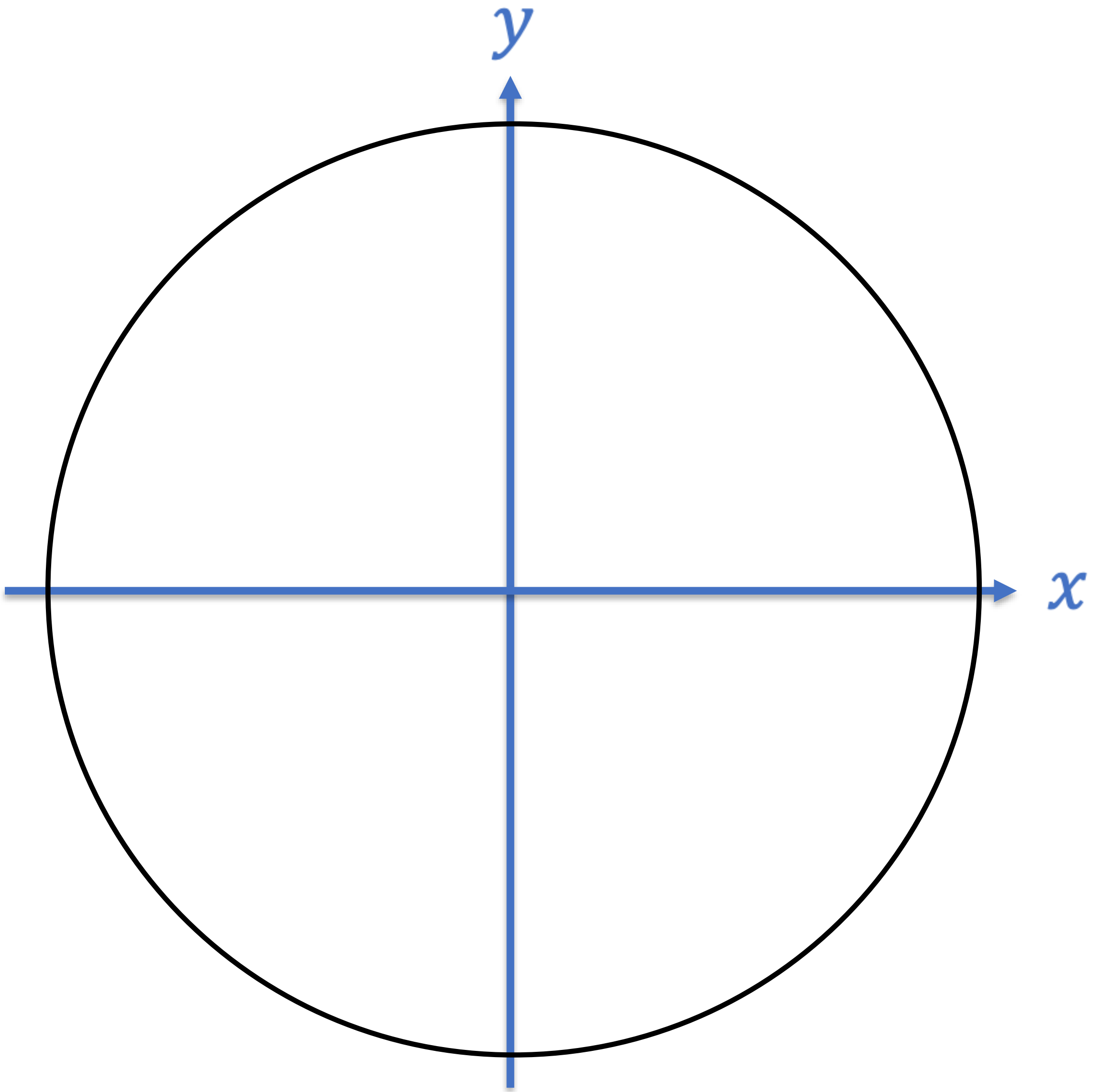


REVIEW



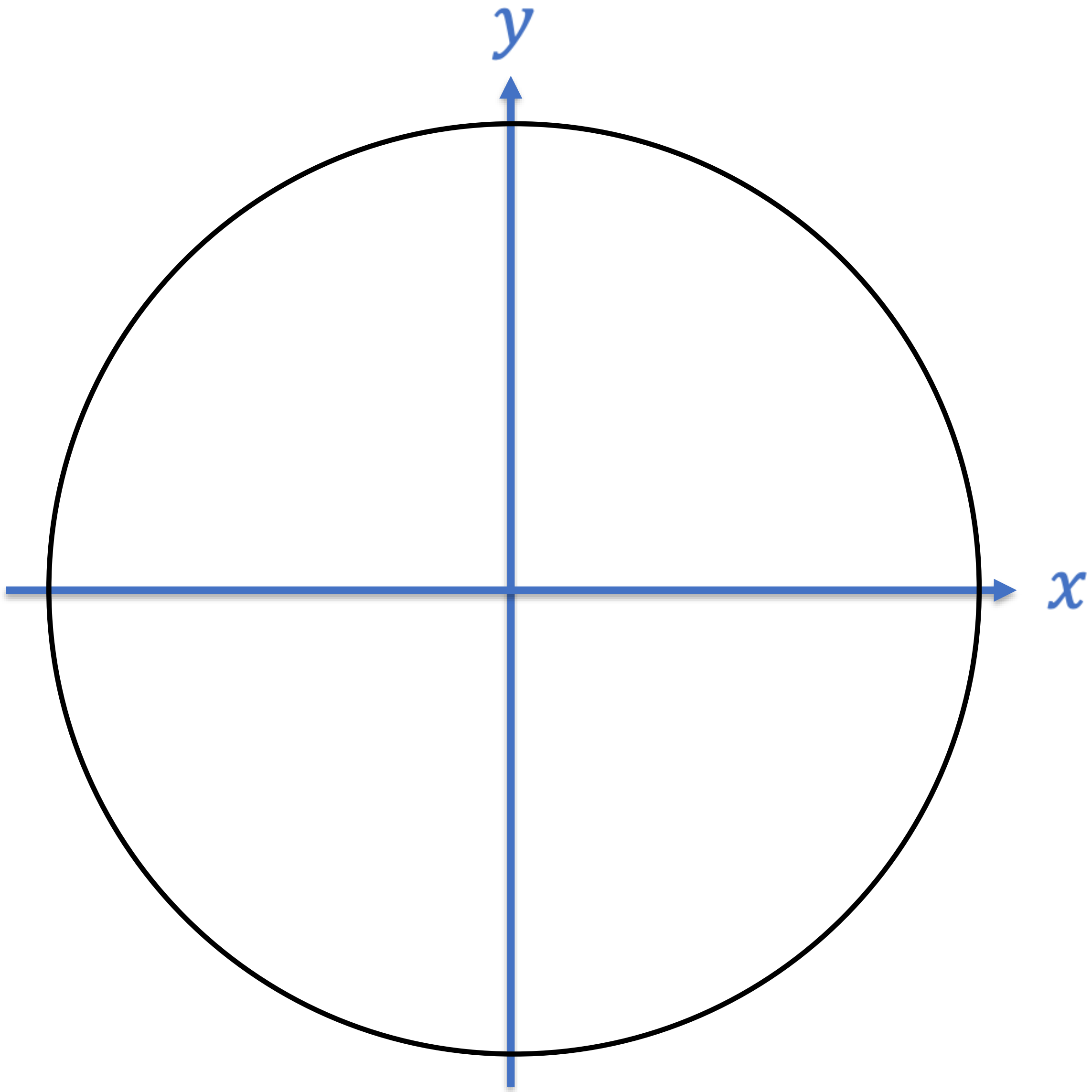


QUESTIONS?



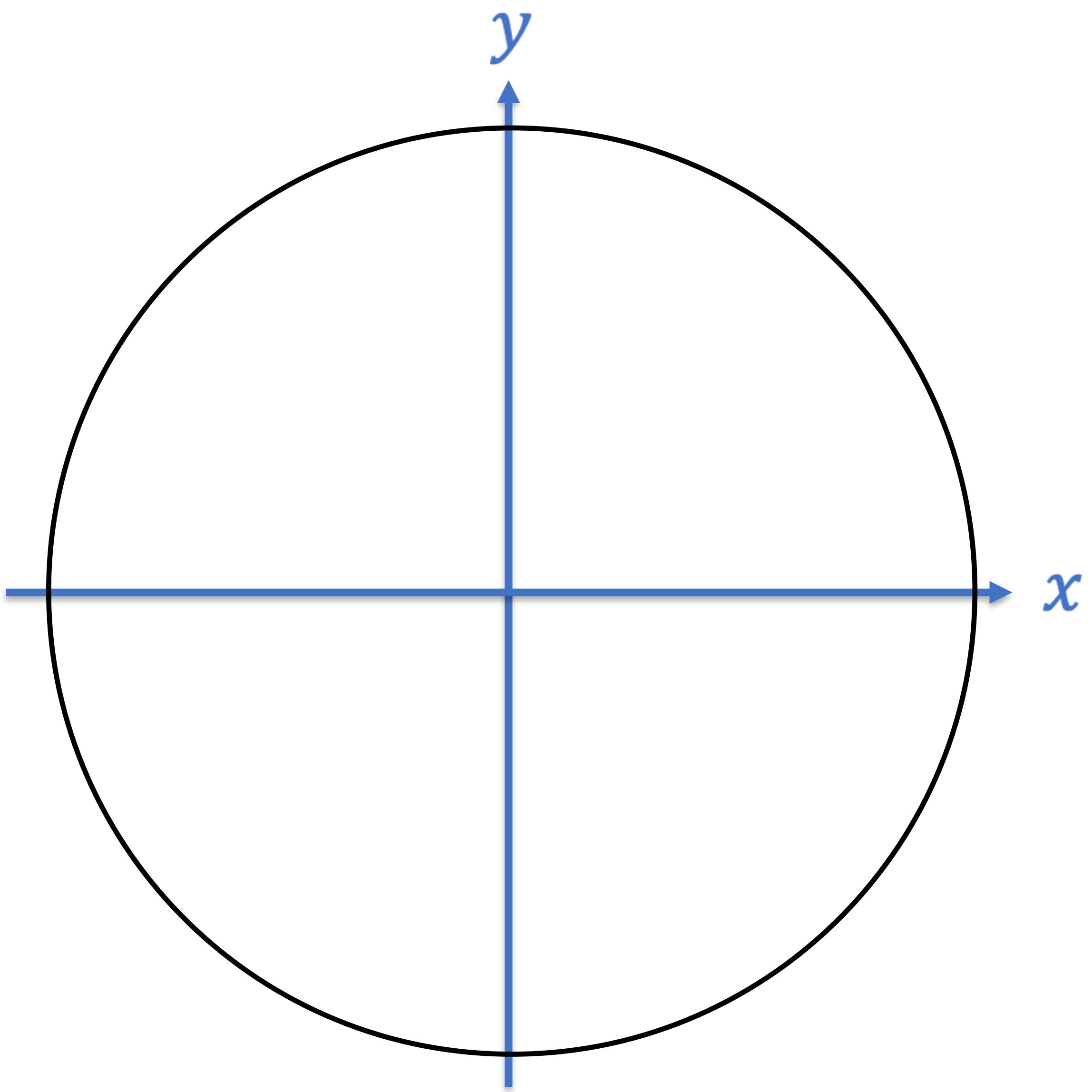
Degrees and radians

WHAT ARE RADIANS?



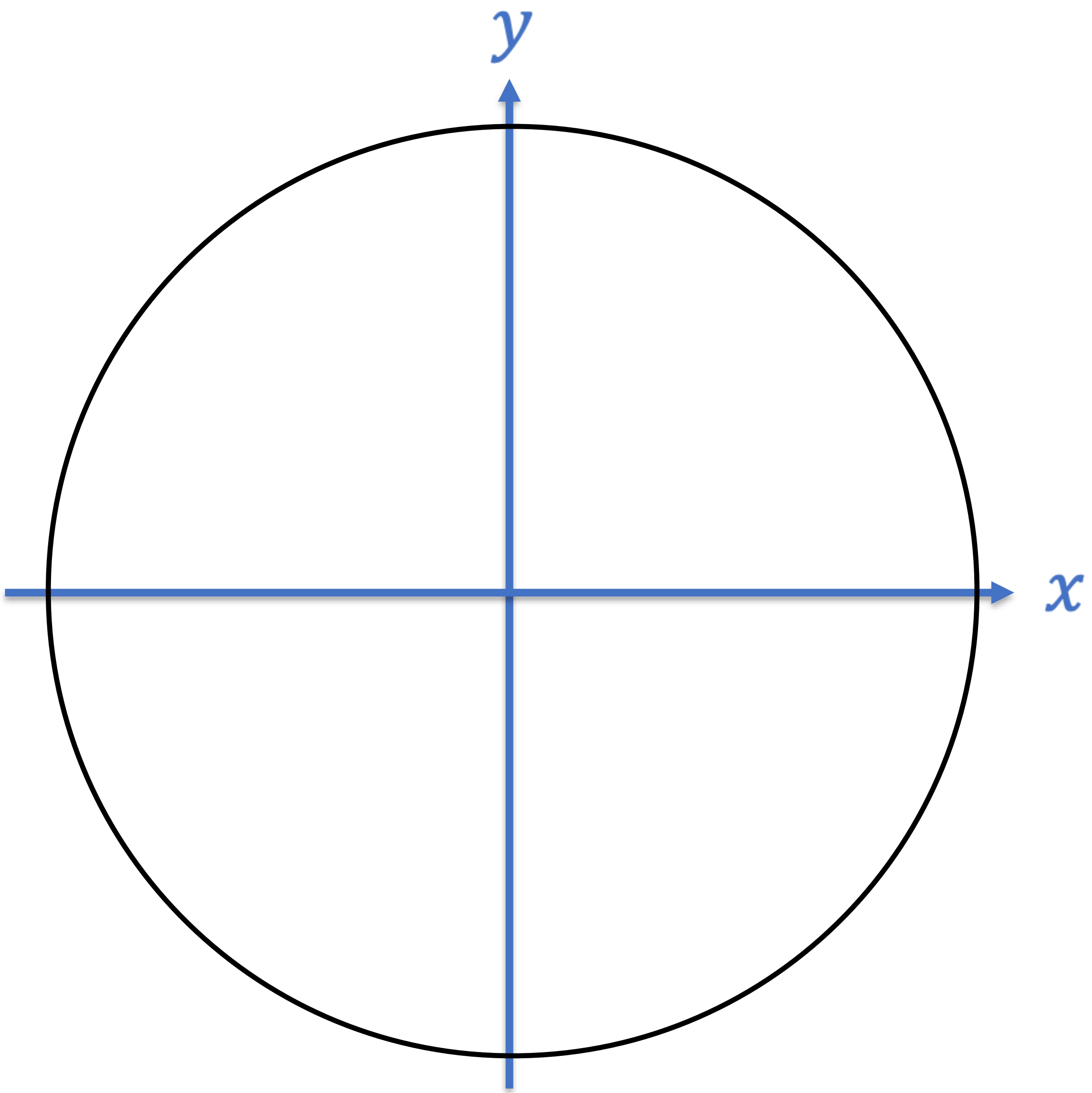
WHY RADIANS?

$$1^\circ = \frac{\pi}{180}\text{rad}$$

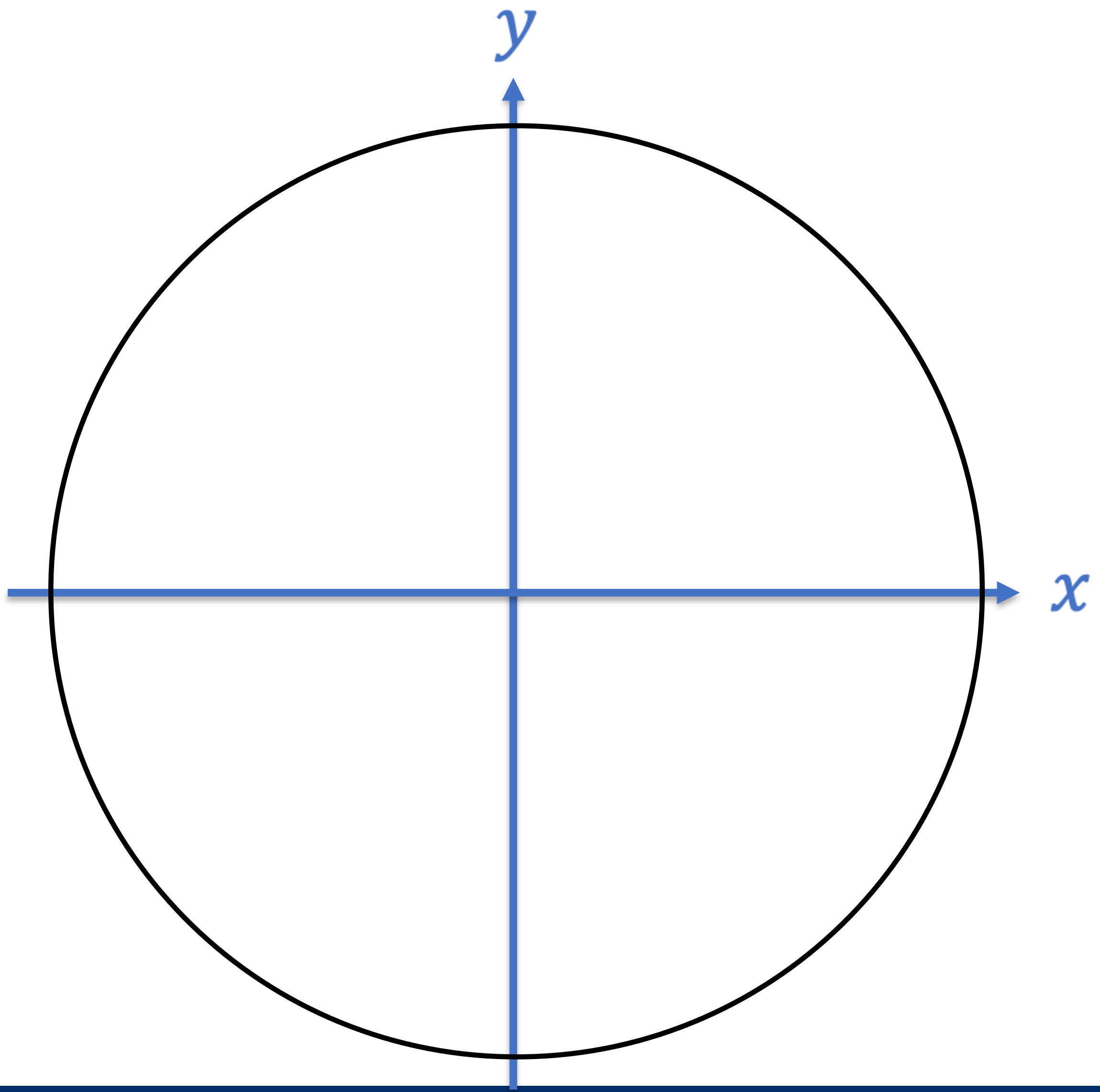


Review

$$1° = \frac{\pi}{180}\text{rad}$$



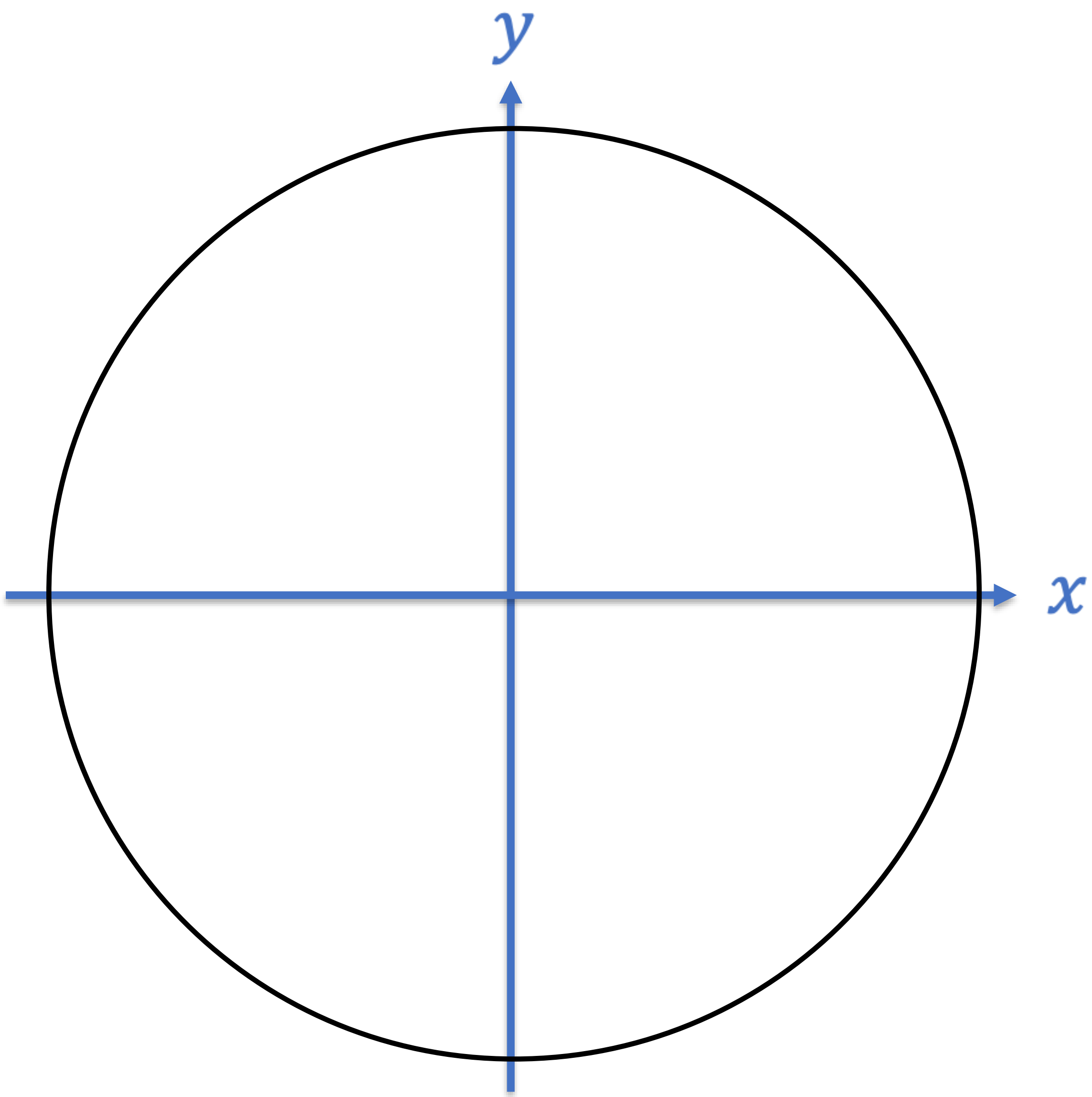
QUESTIONS?



Special angles

SPECIAL ANGLES

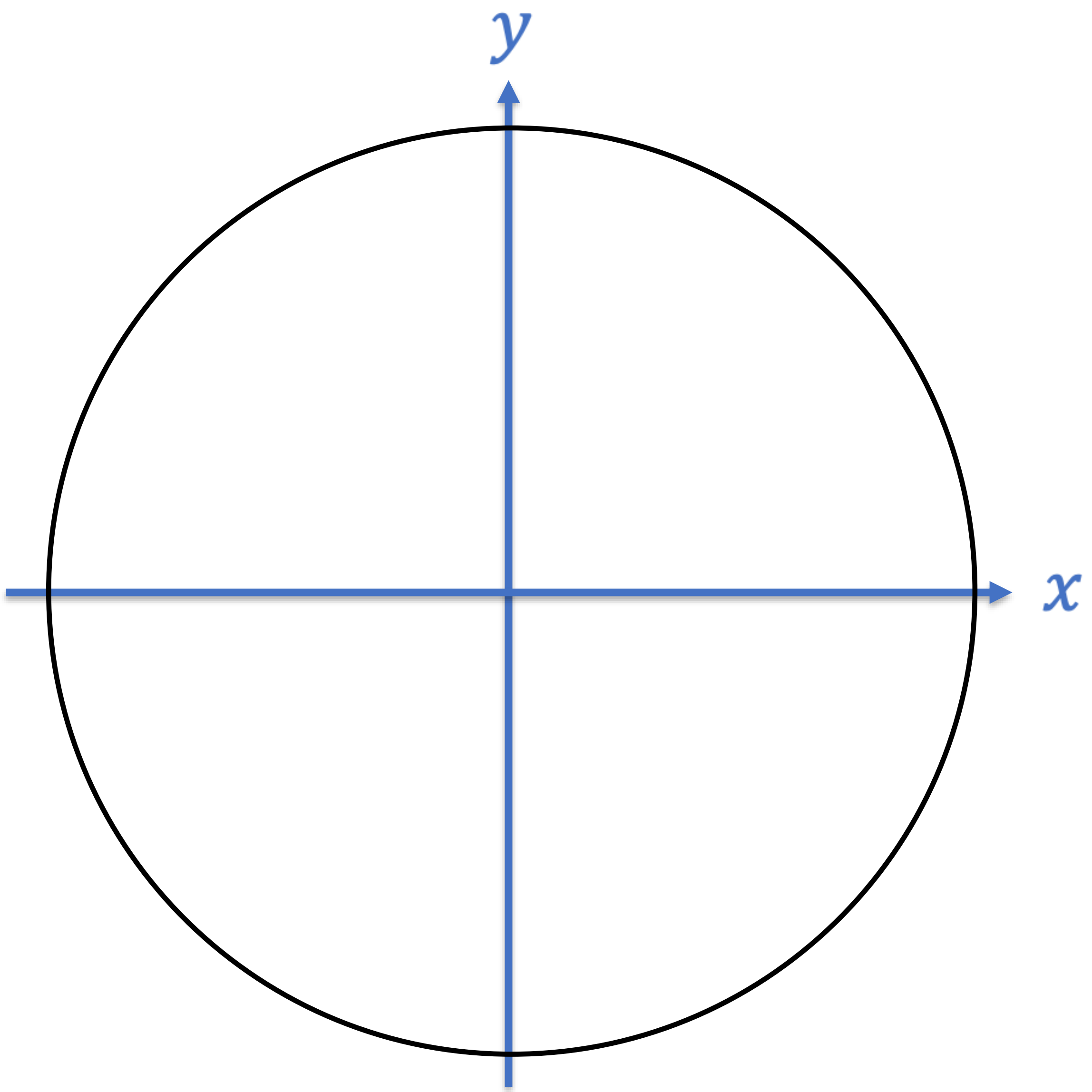
$$1^\circ = \frac{\pi}{180}\text{rad}$$



SPECIAL ANGLES

$$\cos(\mathbf{0}) = \mathbf{1}, \sin(\mathbf{0}) = \mathbf{0}$$

$$\cos\left(\frac{\boldsymbol{\pi}}{\mathbf{2}}\right) = \mathbf{0}, \sin\left(\frac{\boldsymbol{\pi}}{\mathbf{2}}\right) = \mathbf{1}$$

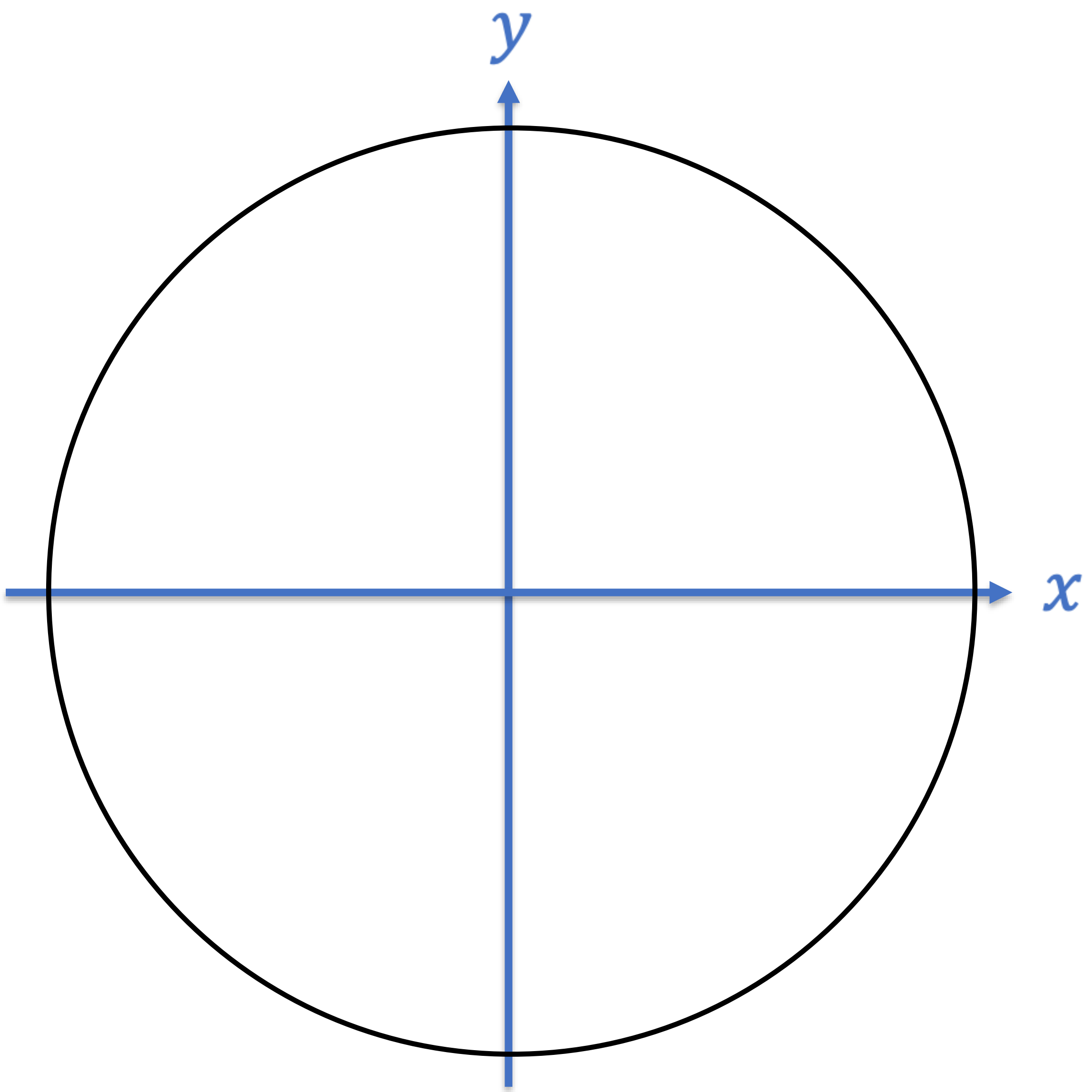


Review

$$\cos(0) = 1, \sin(0) = 0$$

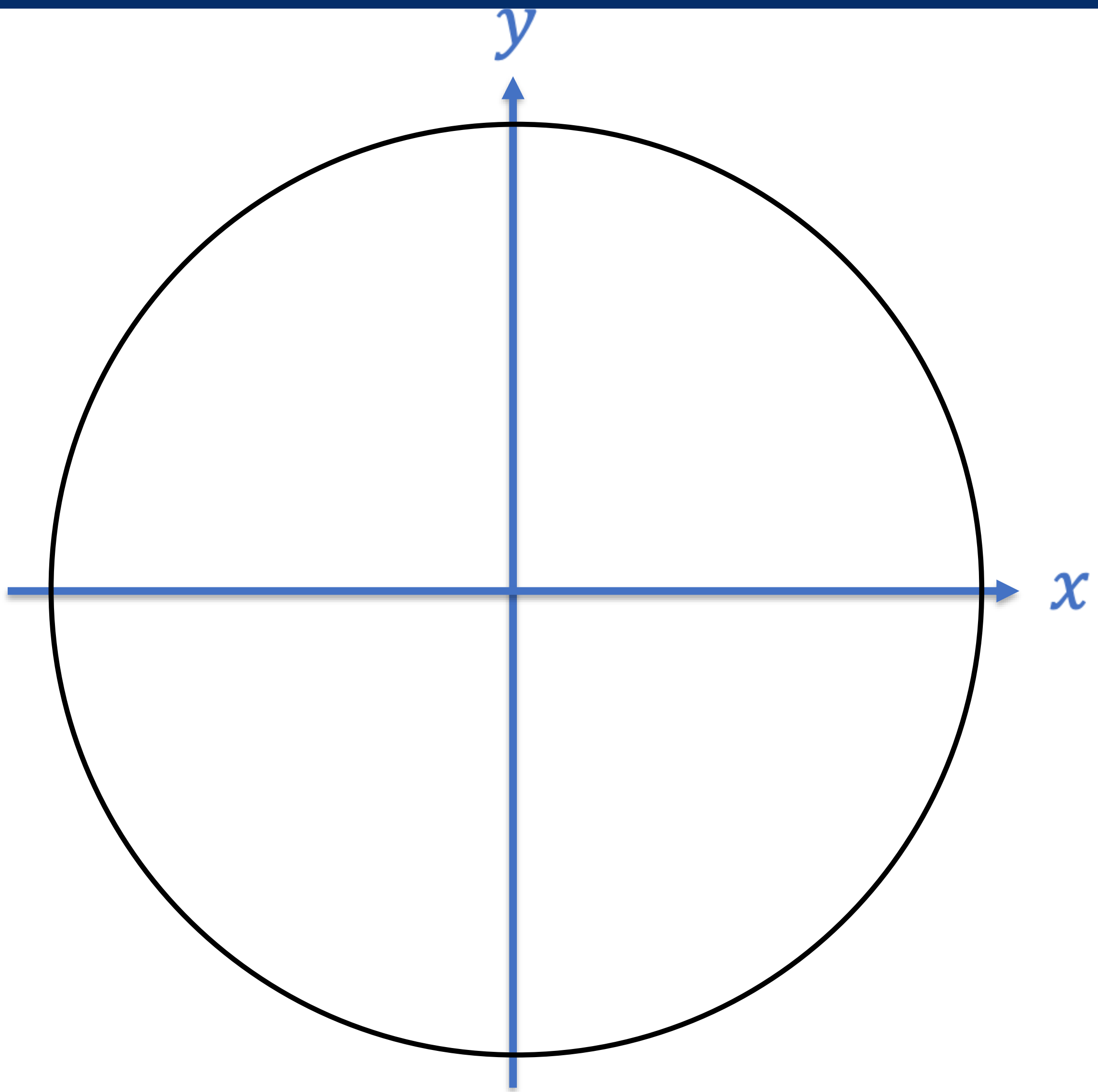
$$\cos\left(\frac{\pi}{2}\right) = 0, \sin\left(\frac{\pi}{2}\right) = 1$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$



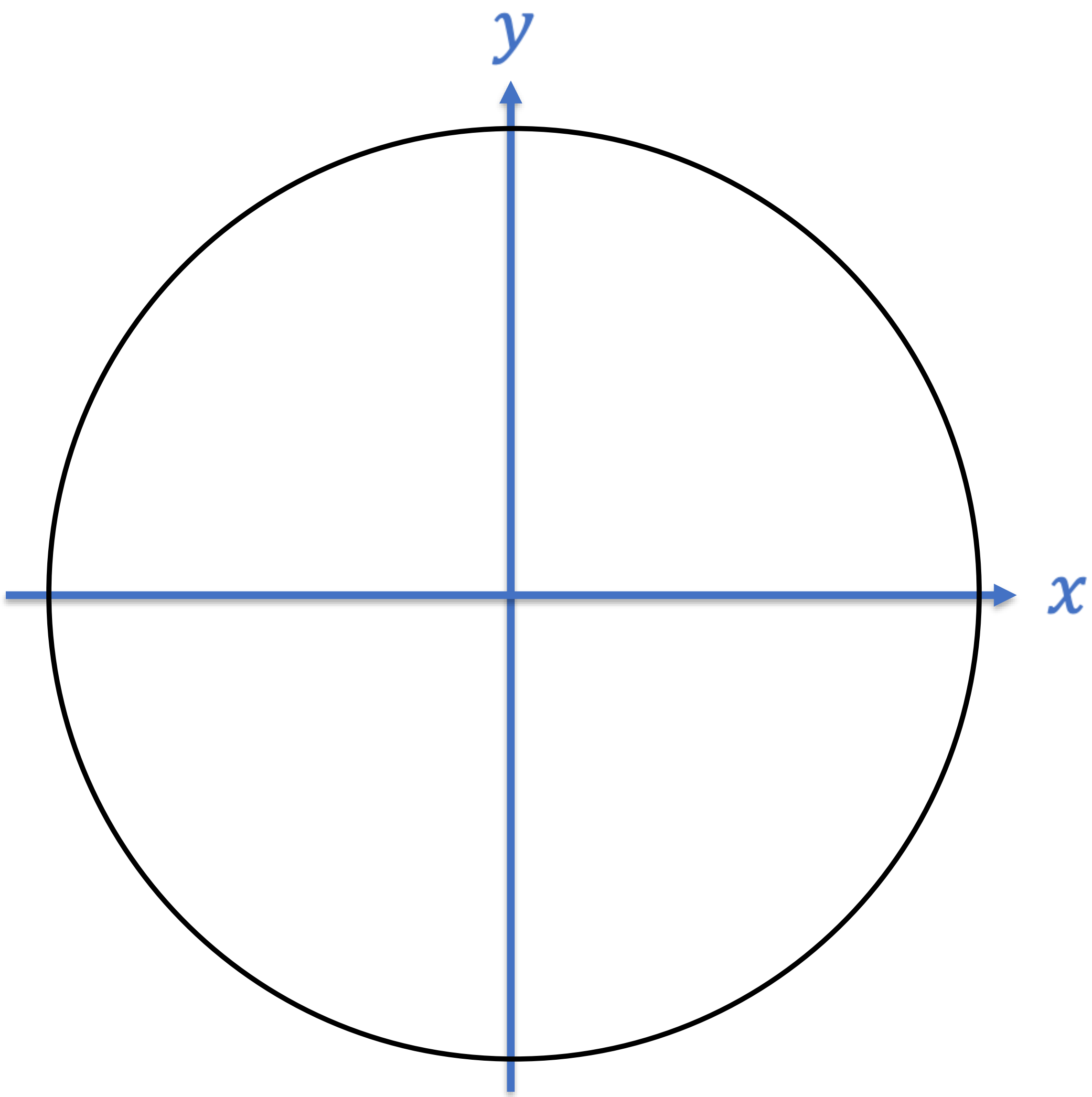
PRACTICE

The point $Q = (c, d)$ lies on the unit circle, and makes a 45° angle with the x axis. Solve for c and d.

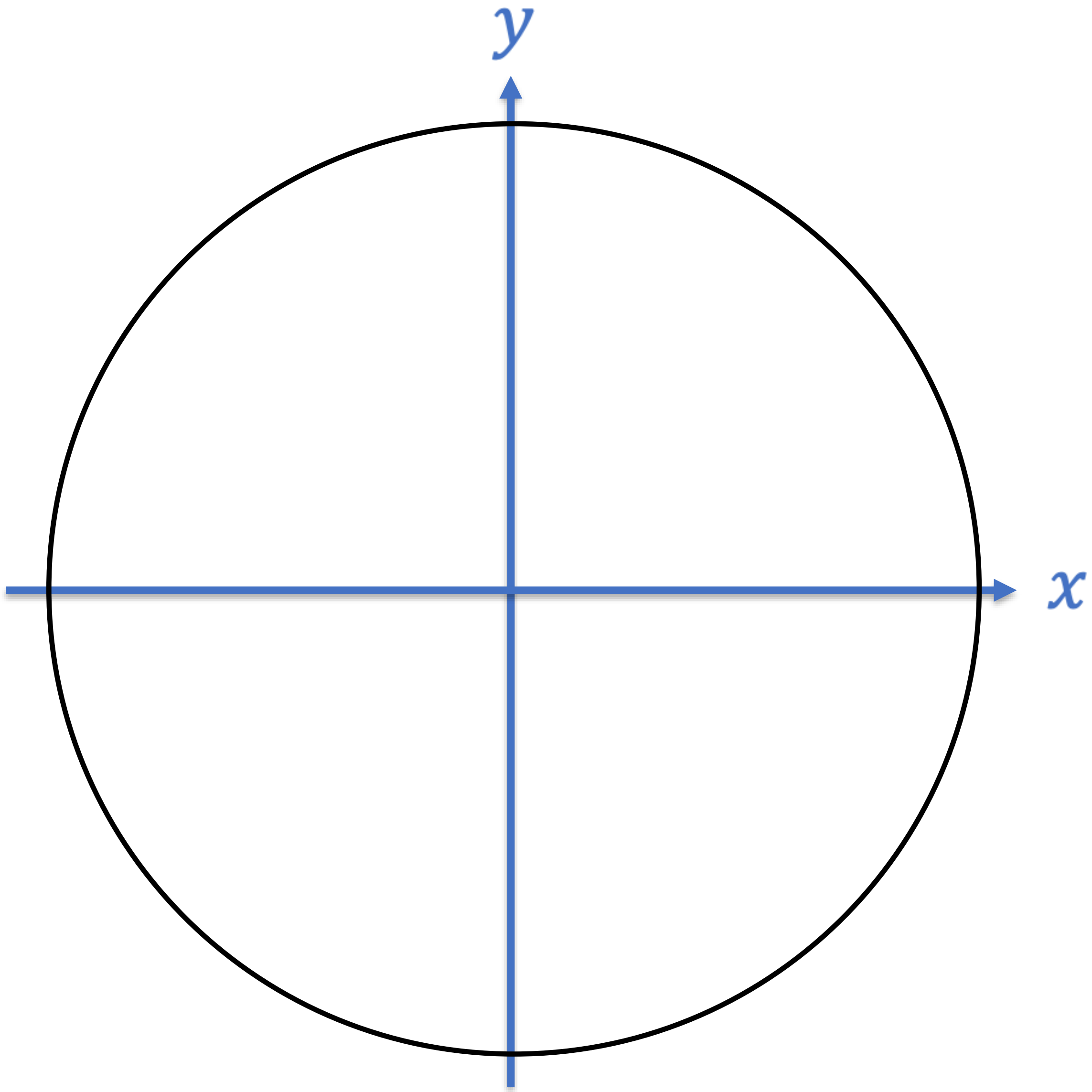


PRACTICE

The point $R = (g, h)$ lies on the unit circle, and makes a 45° angle with the y axis. Solve for g and h.



QUESTIONS?

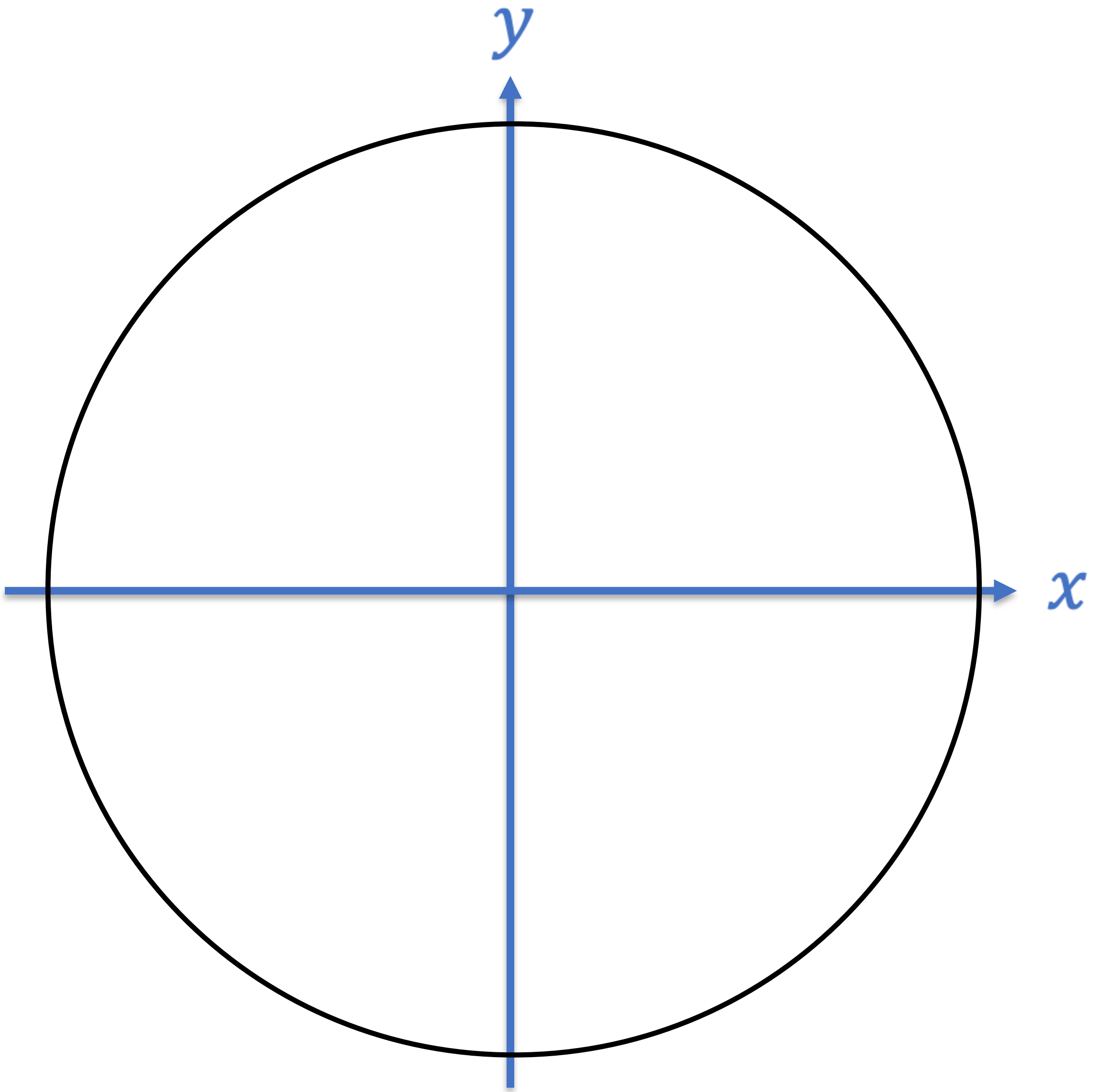


Useful trigonometric identities

USEFUL TRIG IDENTITIES

You are told that $\cos\left(\frac{3\pi}{8}\right) = \frac{1}{2}\sqrt{2-\sqrt{2}}$.

What is $\sin\left(\frac{3\pi}{8}\right)$?

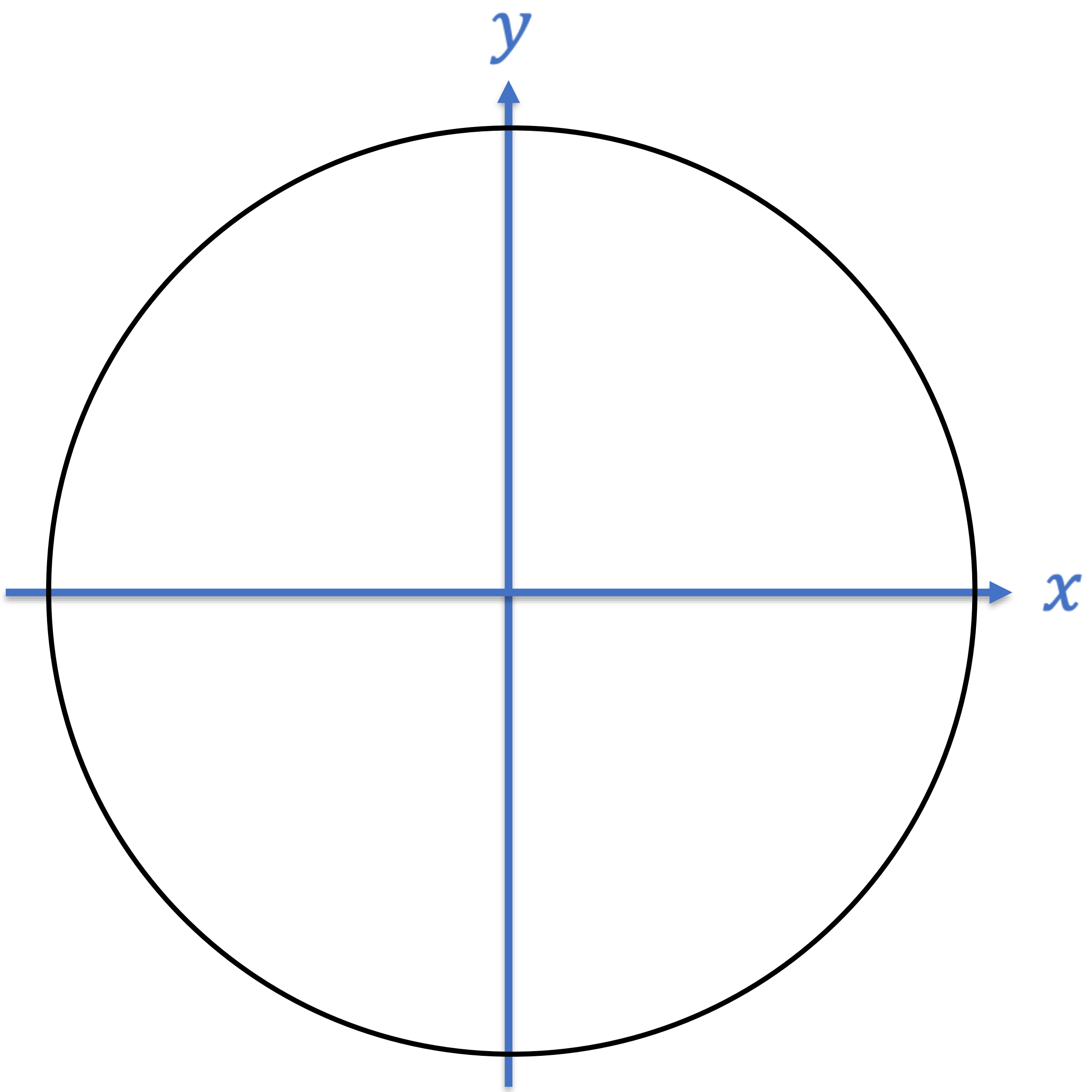


Pythagorean identity:

$$\cos(\boldsymbol{\theta})^2 + \sin(\boldsymbol{\theta})^2 = 1$$

USEFUL TRIG IDENTITIES

Find $\cos\left(-\frac{\pi}{4}\right)$ **and** $\sin\left(-\frac{\pi}{4}\right)$**.**



Negative angles:

$$\cos(-\boldsymbol{\theta}) = \cos(\boldsymbol{\theta})$$

$$\sin(-\boldsymbol{\theta}) = -\sin(\boldsymbol{\theta})$$

USEFUL TRIG IDENTITIES

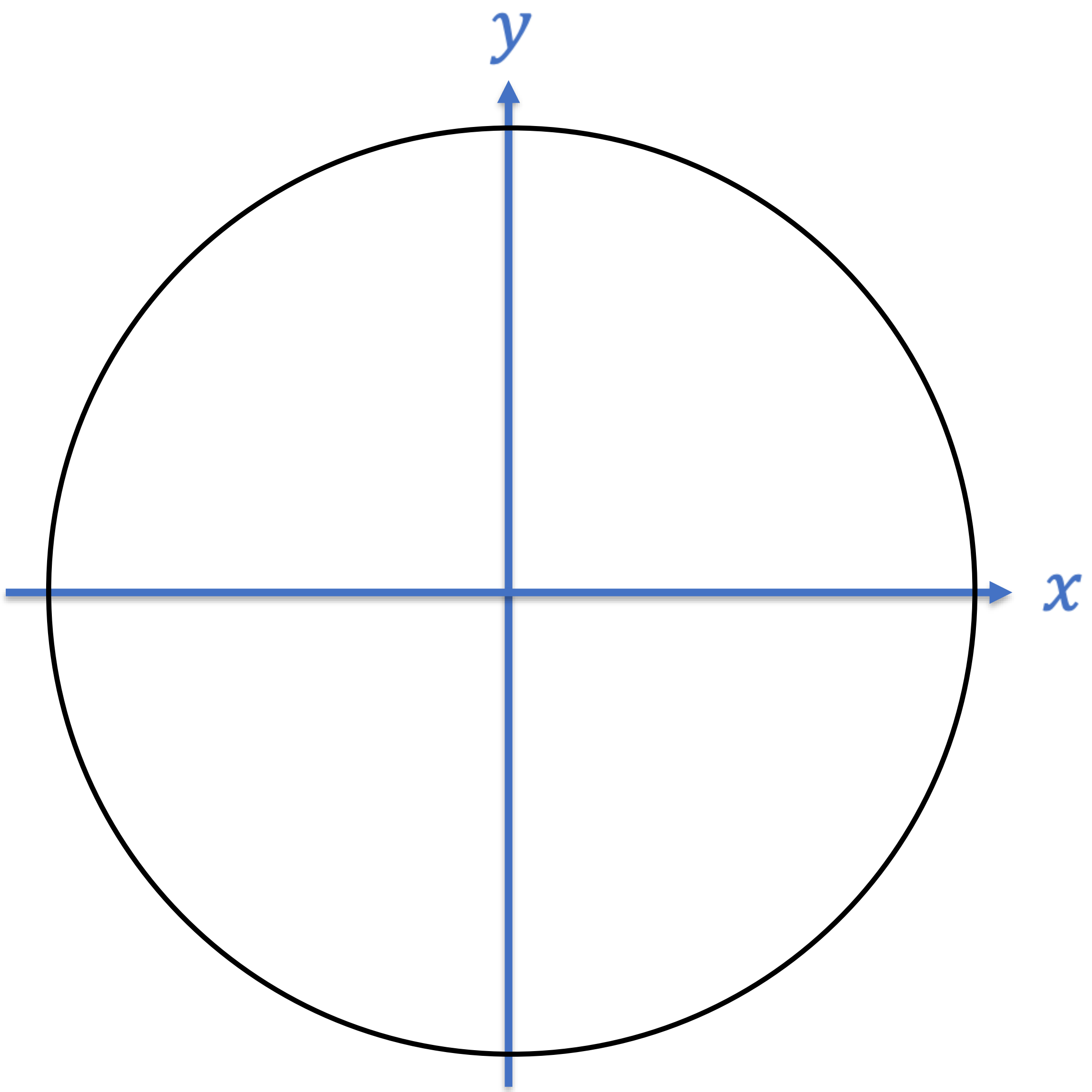
Recall that $\cos\left(\frac{3\pi}{8}\right) = \frac{1}{2}\sqrt{2-\sqrt{2}}$.

Find $\sin\left(\frac{7\pi}{8}\right)$.

$\frac{\pi}{2}$ **shift:**

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos(\theta)$$

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin(\theta)$$



Review

Pythagorean identity:

$$\cos(\boldsymbol{\theta})^2 + \sin(\boldsymbol{\theta})^2 = \mathbf{1}$$

Negative angles:

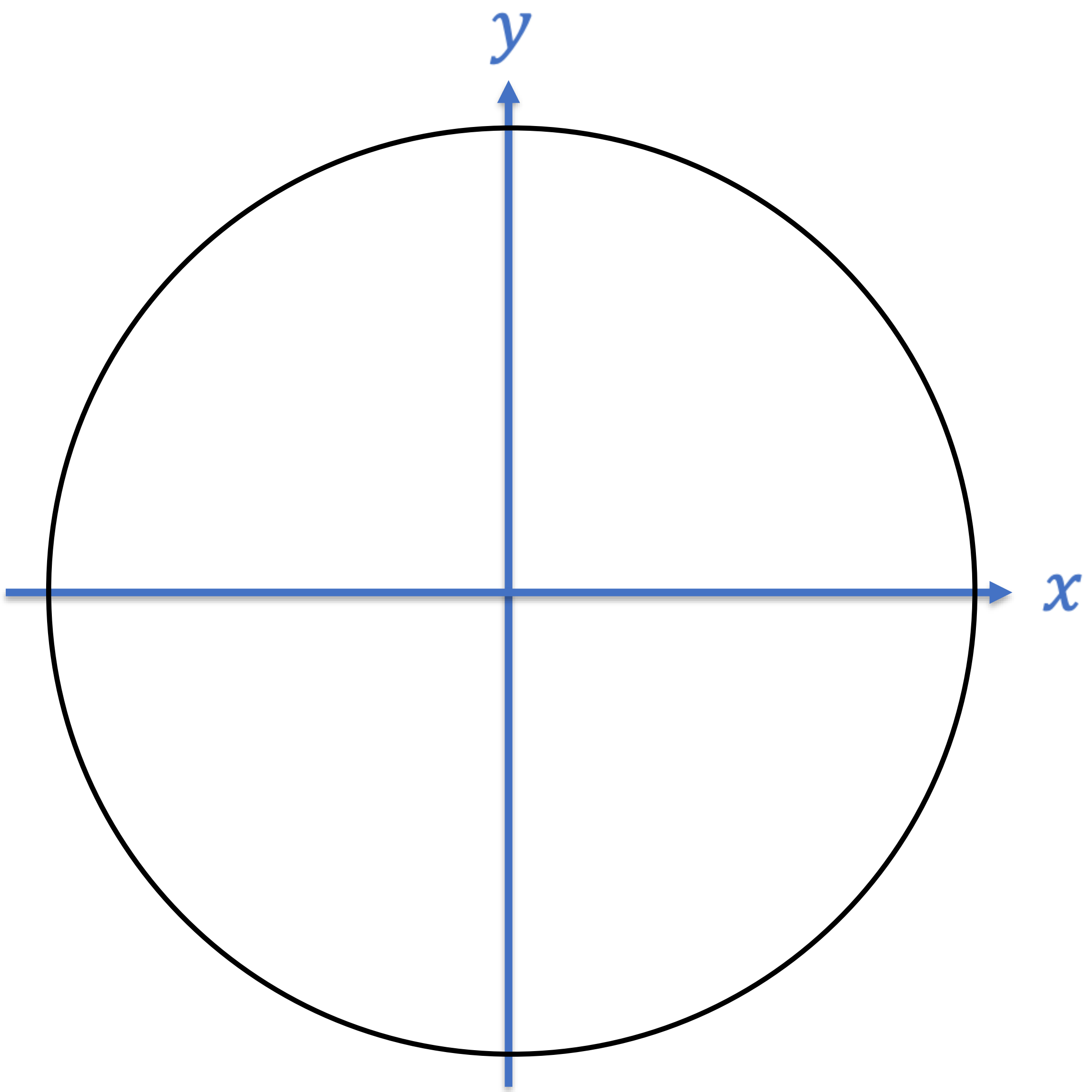
$$\cos(-\boldsymbol{\theta}) = \cos(\boldsymbol{\theta})$$

$$\sin(-\boldsymbol{\theta}) = -\sin(\boldsymbol{\theta})$$

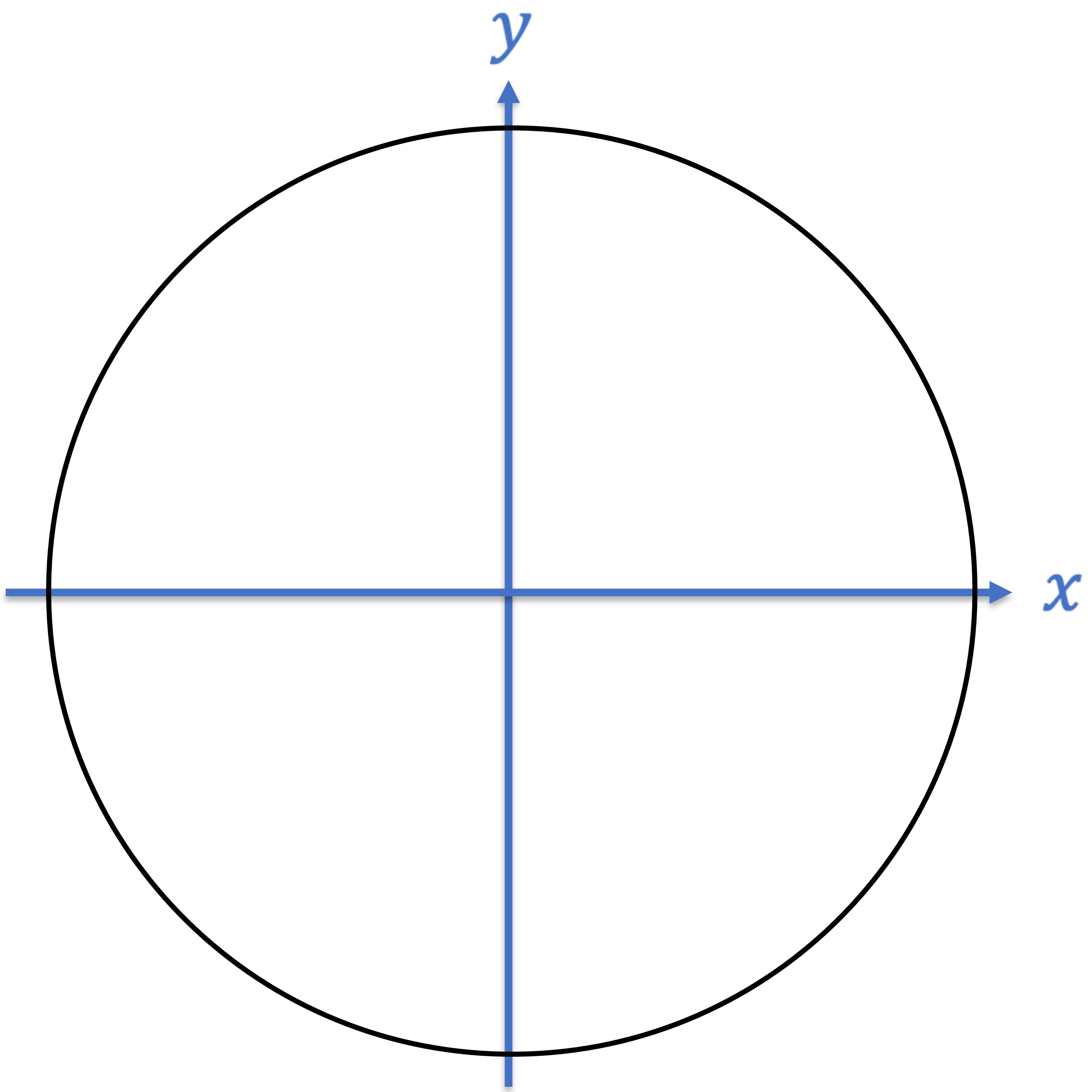
$\frac{\pi}{2}$ **shift:**

$$\sin\left(\boldsymbol{\theta} + \frac{\boldsymbol{\pi}}{\mathbf{2}}\right) = \cos(\boldsymbol{\theta})$$

$$\cos\left(\boldsymbol{\theta} + \frac{\boldsymbol{\pi}}{\mathbf{2}}\right) = -\sin(\boldsymbol{\theta})$$

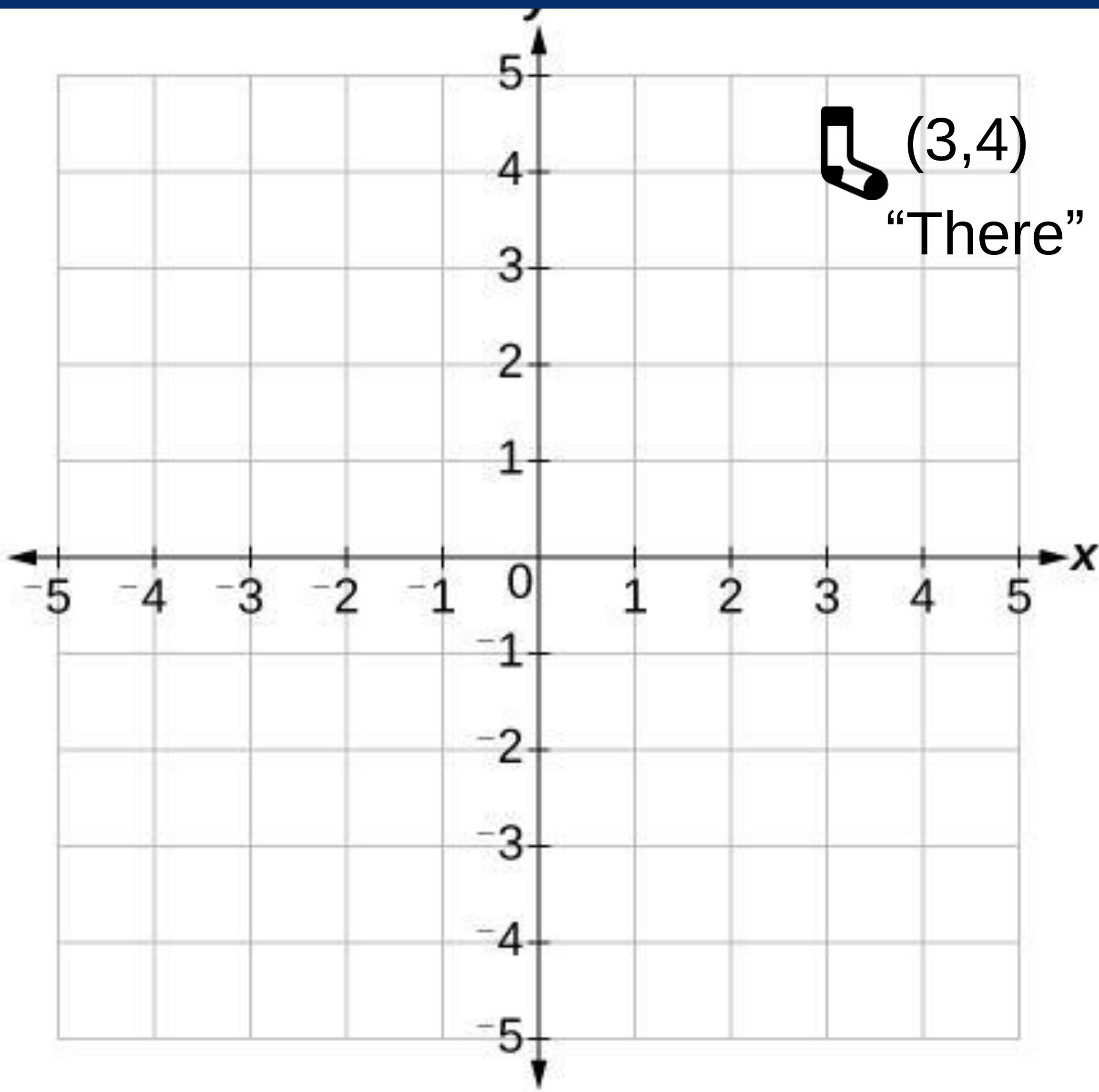


QUESTIONS?

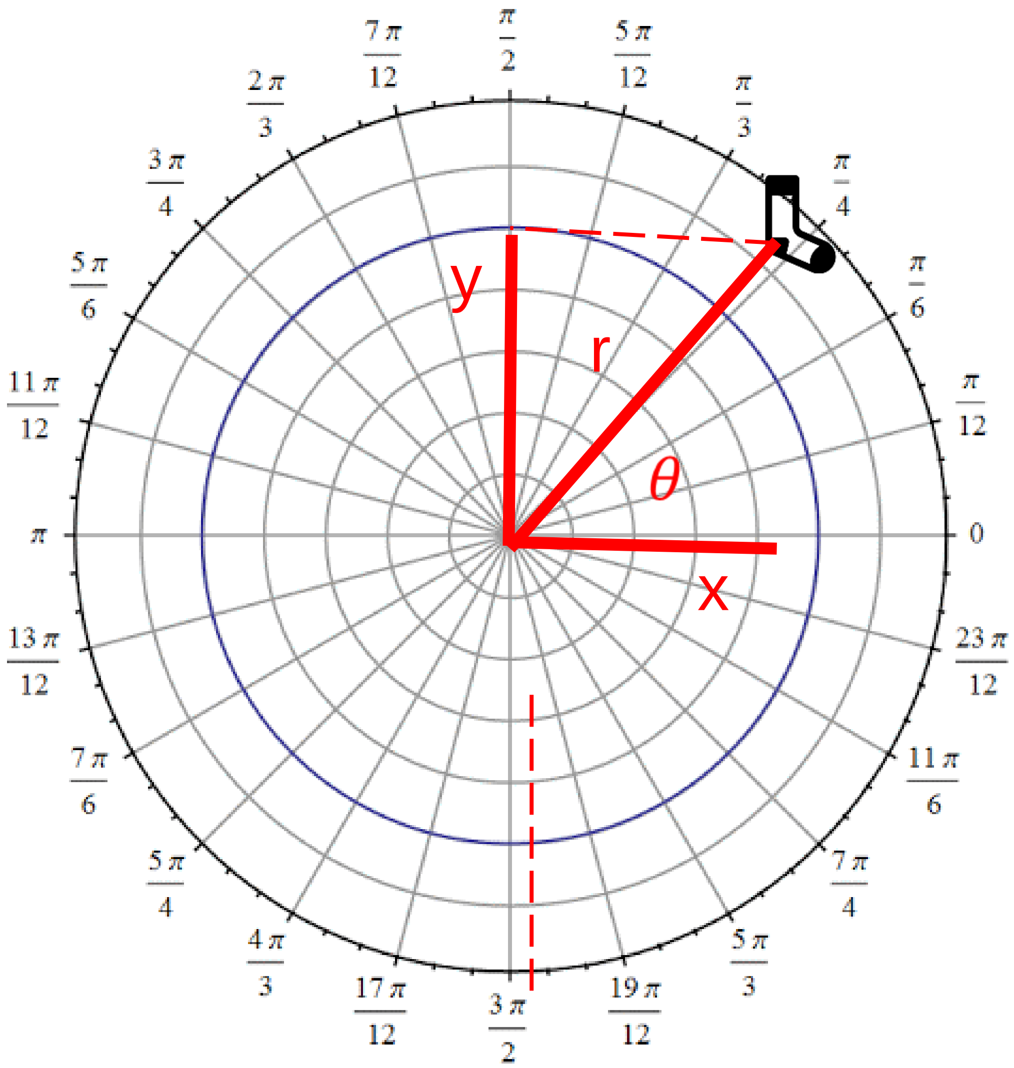


Coordinate systems

COORDINATES



ENTER SINELAND-1A POLAR COORDINATES



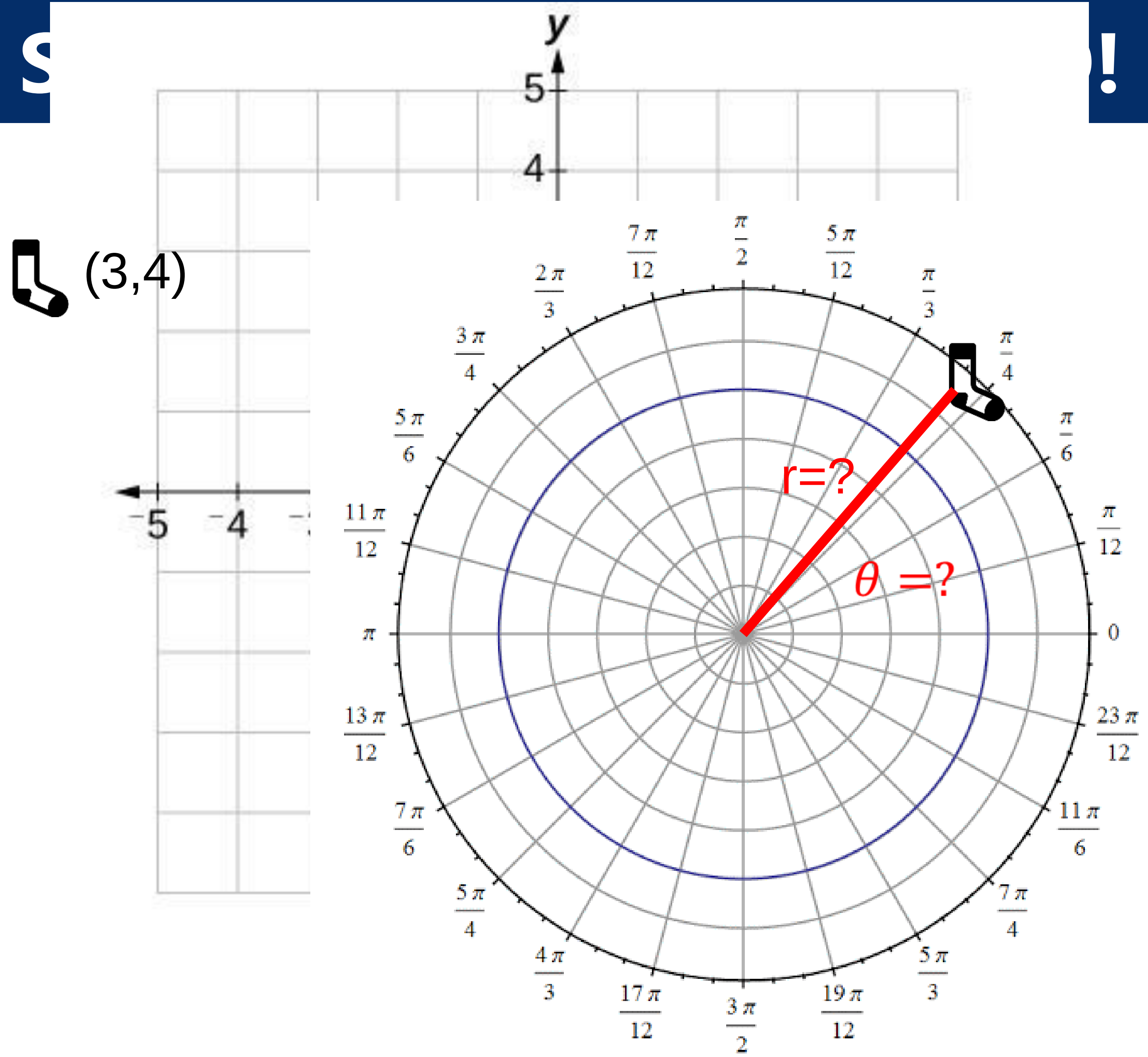
$x = ?$

$y = ?$

$x = r\cos\theta$

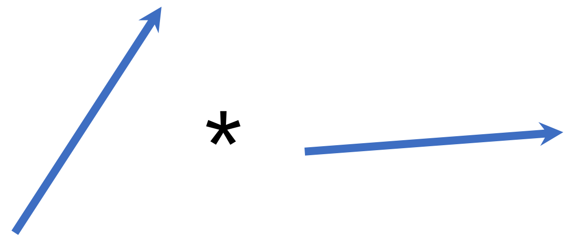
$y = r\sin\theta$

LOCATE THE S ... !

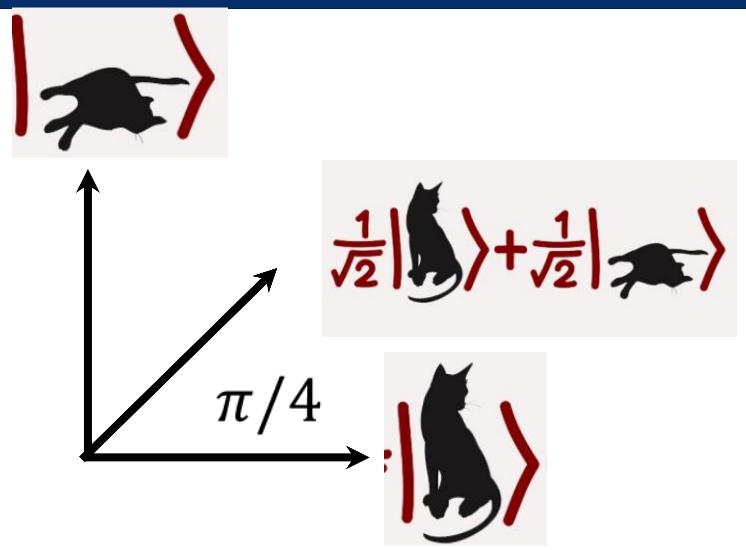


(3,4)
y
5
4
-5
-4
r=?
θ =?
π/2
5π/12
π/3
π/4
π/6
π/12
0
23π/12
11π/6
7π/4
5π/3
19π/12
3π/2
17π/12
4π/3
5π/4
7π/6
13π/12
π
11π/12
5π/6
3π/4
2π/3
7π/12

VECTORS, QM STATES, COMPLEX NUMBERS!



Operations between vectors are easier with their components!



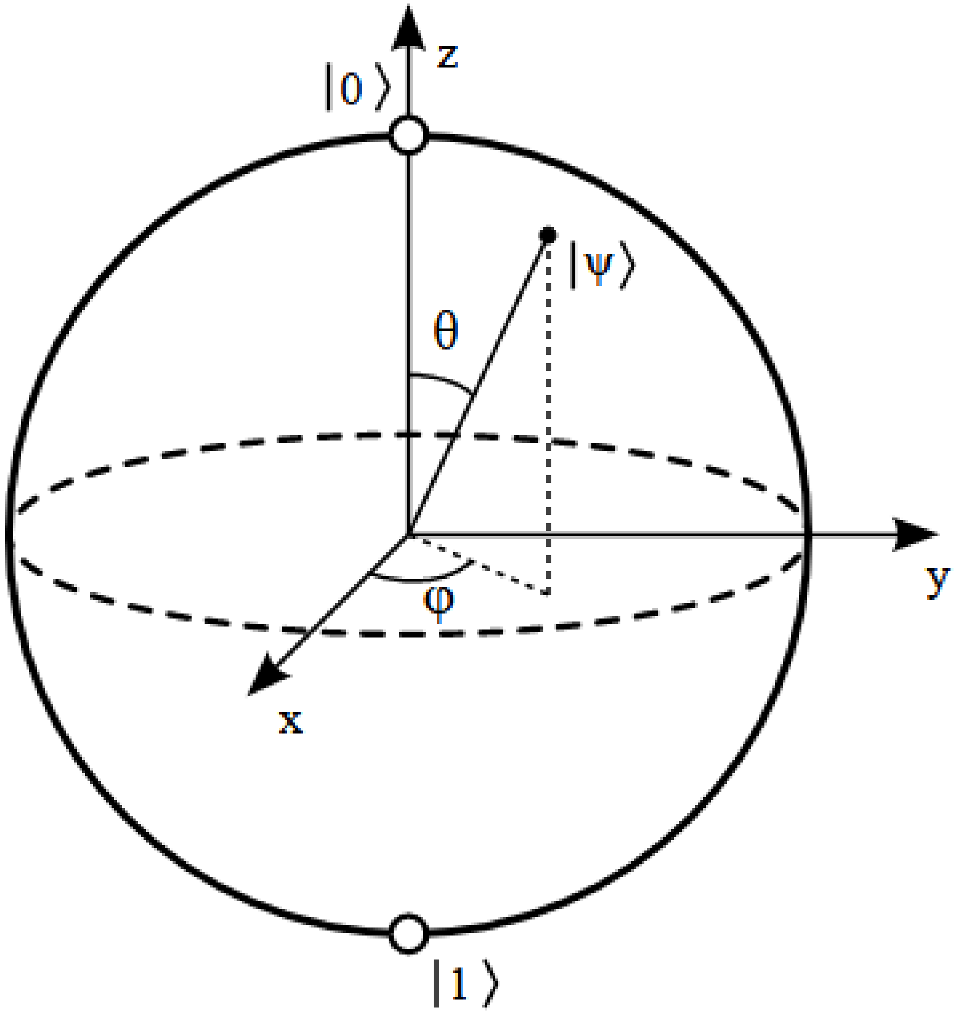
QM States are vectors in Hilbert Space.

Complex Numbers
In Future!

Complex numbers have a super useful polar form which makes everything intuitive and easy!

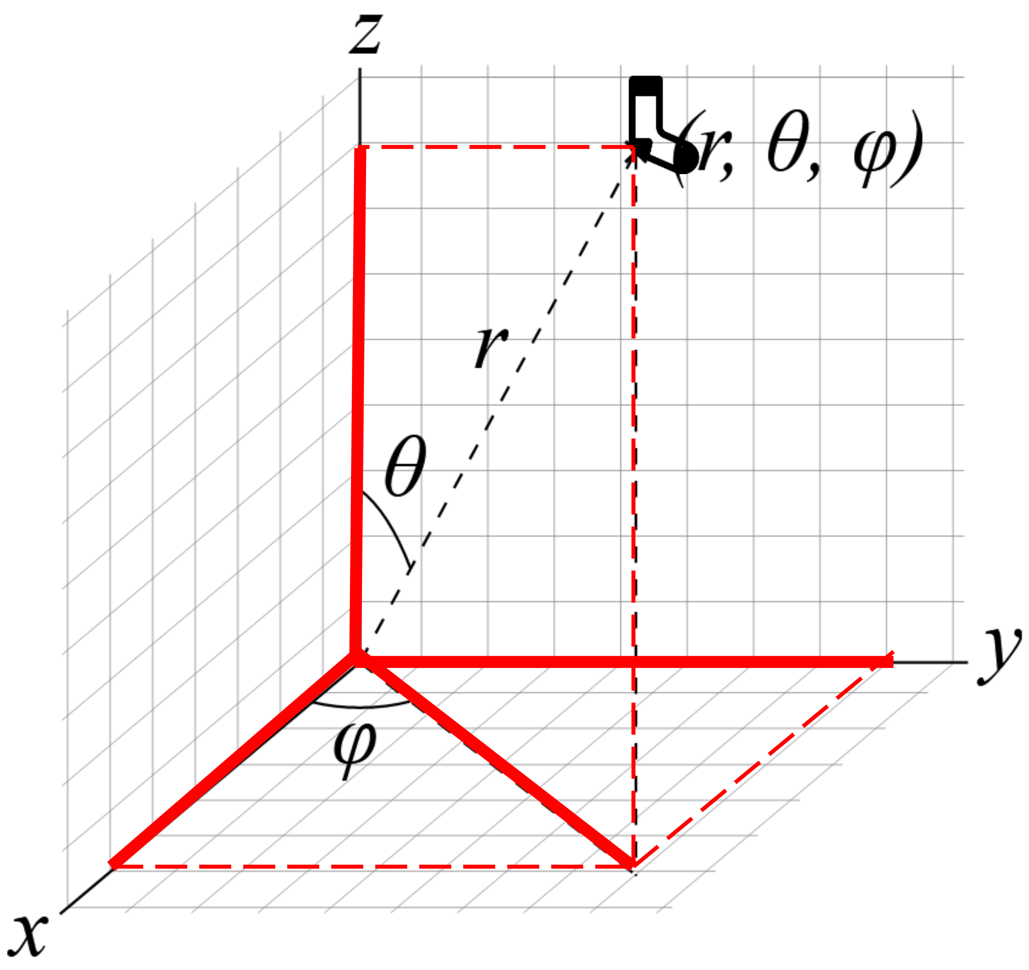
QUESTIONS

THE BLOCH SPHERE!



Quantum states in quantum computing are often represented on a Bloch spheres and operations in quantum computing amounts to moving the state around.

ENTER SINELAND-1B SPHERICAL COORDINATES



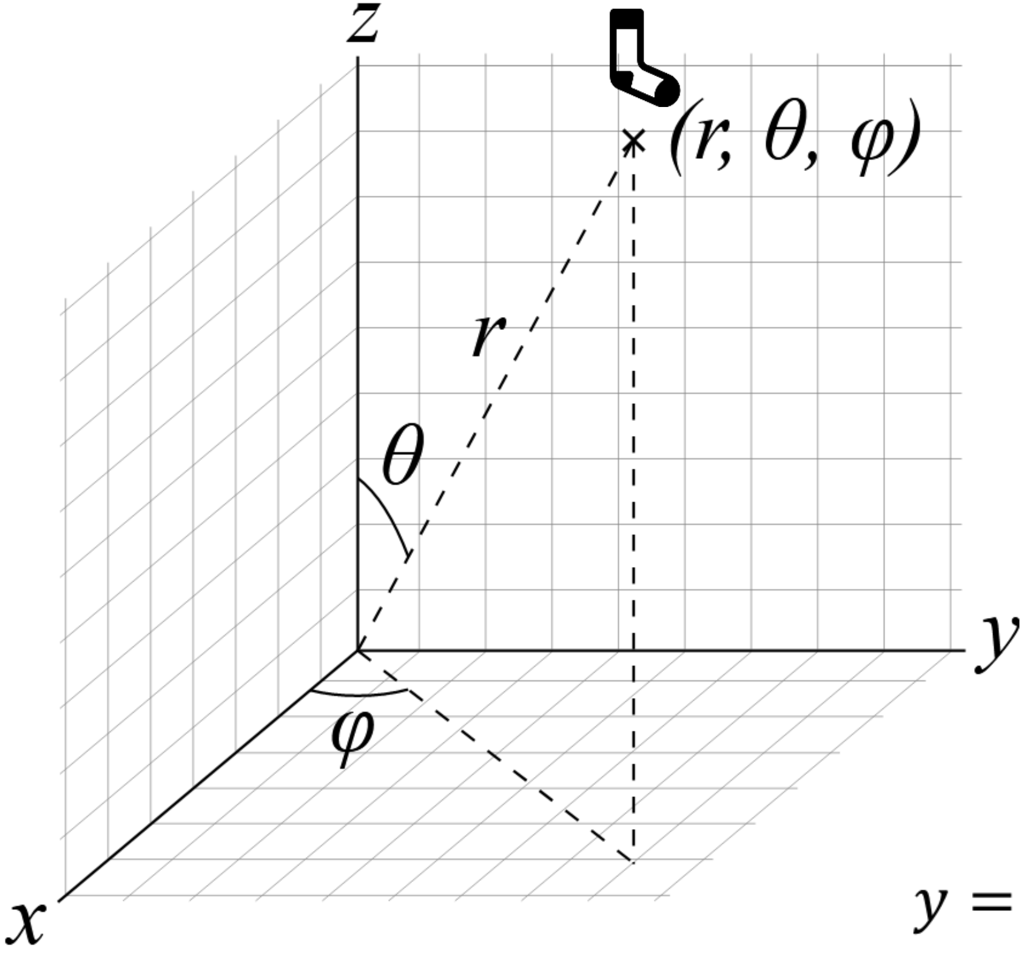
Watch out!
Mathematicians use different convention than Physicists and switch θ and φ

$z = ?$ $\quad z = r\cos\theta$

$x = ?$ $\quad x = r\sin\theta\cos\varphi$

$y = ?$ $\quad y = r\sin\theta\sin\varphi$

TELL YOUR CARTESIAN FRIEND TO FIND SOCK!



$$(r, \theta, \phi) = (2, \pi/4, \pi/4)$$

$z = ?$

$x = ?$

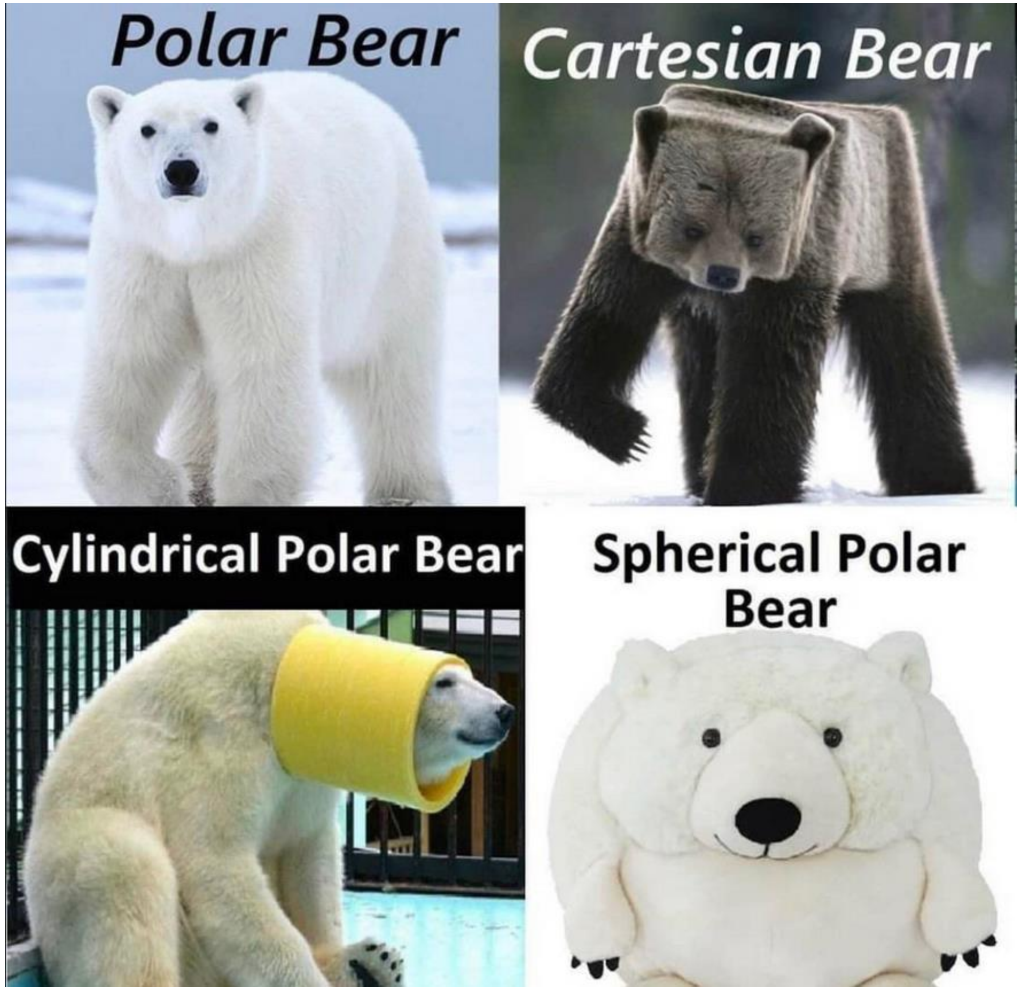
$y = ?$

$$z = 2\cos(\pi/4) = 2\frac{1}{\sqrt{2}} = \sqrt{2}$$

$$x = 2\sin(\pi/4)\cos(\pi/4) = 2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} =$$

$$y = 2\sin(\pi/4)\sin(\pi/4) = 2\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} = 1$$

QUESTIONS?


Polar Bear
Cartesian Bear
Cylindrical Polar Bear
Spherical Polar Bear

Waves

ENTER SINELAND 2: WAVES

Quantum Mechanics: Wave Matter Duality!
de Broglie:
A particle moving with momentum p has a wavelength!

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Planck's constant $h = 6.626 \times 10^{-34} m^2 kg/s$

Q: Why do moving objects around us not appear wavy?

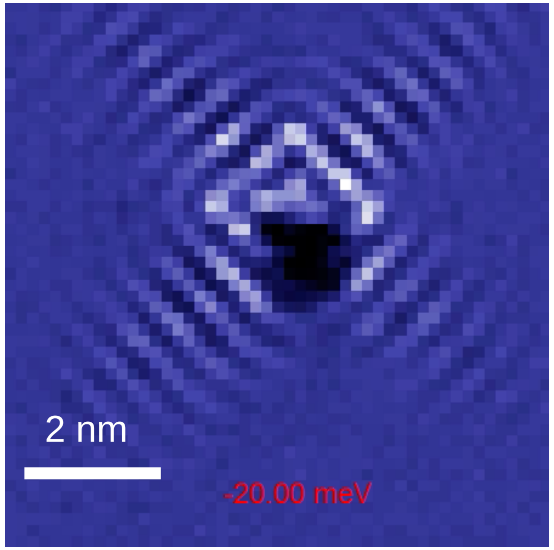
For instance, let us take golf ball: ~ 46g moving at 1 m/s

$$\lambda = ?$$

$$\lambda \approx 10^{-32} m!$$

How about an electron ($m \approx 10^{-30}$) moving at 10^6 m/s?

$\lambda \approx 10^{-10}$m ~ size of an atom!



ENTER SINELAND 2: WAVES

$\sin(2\pi + x) = \sin(x)$, Trigonometrical functions are periodic

$$\psi(x) = A\sin(kx + \phi)$$

A: Amplitude

k: Wave-vector

ϕ: Phase

Let us explore this sineland in Geogebra!

QUESTIONS?